

ENGINEERING ADMISSIONS ASSESSMENT

D564/11

Wednesday 2 November 2016

80 minutes

SECTION 1

INSTRUCTIONS TO CANDIDATES

Please read these instructions carefully, but do not open this question paper until you are told that you may do so. This paper is Section 1 of 2.

A separate answer sheet is provided for this paper. Please check you have one. You also require a soft pencil and an eraser.

Please complete the answer sheet with your candidate number, centre number, date of birth, and name.

At the end of 80 minutes, your supervisor will collect this question paper and answer sheet before giving out Section 2.

This paper contains **two** parts, **A** and **B**, and you should attempt **both** parts.

Part A Mathematics and Physics (28 questions)

Part B Advanced Mathematics and Advanced Physics (26 questions)

This paper contains 54 multiple choice questions. There are no penalties for incorrect responses, only marks for correct answers, so you should attempt all 54 questions. Each question is worth one mark.

Questions ask you to show your choice between options. Choose the **one** option you consider correct and record your choice on the separate answer sheet. If you make a mistake, erase thoroughly and try again.

You can use the question paper for rough working, but no extra paper is allowed. Only your responses on the answer sheet will be marked.

Dictionaries and calculators may NOT be used.

Please wait to be told you may begin before turning this page.

This question paper consists of 44 printed pages and 4 blank pages.

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PART A Mathematics and Physics

- 1 Find the complete set of solutions to $-8 < 6 - \frac{x}{2}$

A $x < 4$

B $x > 4$

C $x < 20$

D $x > 20$

E $x < 22$

F $x > 22$

☒ G $x < 28$

H $x > 28$

$$-8 < 6 - \frac{x}{2}$$

$$-14 < -\frac{x}{2}$$

$$x > 28$$

- 2 A nuclide $^{214}_{82}\text{Pb}$ changes by radioactive decay into the nuclide $^{210}_{82}\text{Pb}$.
Which combination of emissions produces this change?

A 3 alpha

B 2 alpha and 1 beta

C 2 alpha and 2 beta

☒ D 1 alpha and 2 beta

E 3 beta

$$\alpha \rightarrow 214 \text{ goes to } 210$$

$$82 \text{ goes to } 80$$

$$\therefore \text{We need } 2 \alpha \text{ } \beta^-$$

- 3 Which one of the following is a simplification of $(\sqrt{3} - \sqrt{2})^2$?

A $1 - 2\sqrt{3}\sqrt{2}$

$$= (\sqrt{3} - \sqrt{2})(\sqrt{3} - \sqrt{2})$$

B $5 - 2\sqrt{2}\sqrt{3}$

$$= 3 - \sqrt{6} - \sqrt{6} + 2$$

C $2\sqrt{3} - 2\sqrt{2}$

$$= 5 - 2\sqrt{2}\sqrt{3}$$

D 1

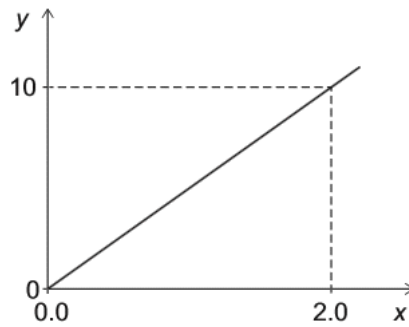
E $5 - \sqrt{2}\sqrt{3}$

F $13 - 2\sqrt{2}\sqrt{3}$

G $5 + 2\sqrt{2}\sqrt{3}$

H 5

- 4 The graph shown of quantity y against quantity x represents the motion of a body.



(The scales on both axes are in the appropriate S.I. units, and the gravitational field strength g is 10 N kg^{-1} .)

Which two of the following could the graph represent?

- 1 kinetic energy against velocity for an object of mass 10 kg undergoing free-fall $KE \propto v^2$
- 2 potential energy against height for an object of mass 20 kg being lifted by a constant external force $W \propto h$ but $20g \times 2 \neq 10$
- 3 ✓ velocity against time for an object of mass 20 kg being accelerated by a resultant force of 100 N $v = 0 + \frac{100}{20}t = 5t$
- 4 ✓ work done by an external force of 5 N against distance moved for an object of mass 12 kg being moved at constant speed by (and in the direction of) the external force $W = 5 \times 2 = 10$

A 1 and 2

B 1 and 3

C 1 and 4

D 2 and 3

E 2 and 4

F 3 and 4

- 5 The ratio of Q:R is 5:2 and the ratio of R:S is 3:10

Which one of the following gives the ratio Q:S in its simplest form?

A 1:2

B 2:1

☒ C 3:4

D 3:25

E 4:3

F 25:3

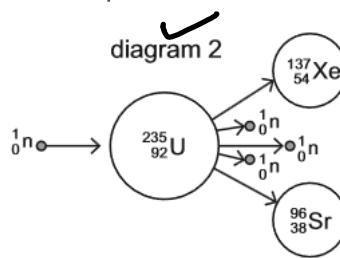
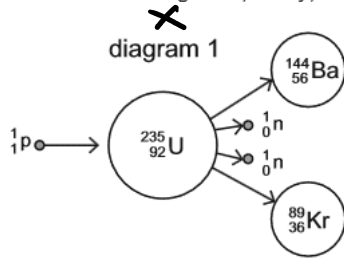
$$Q:R = 15:6$$

$$R:S = 6:20$$

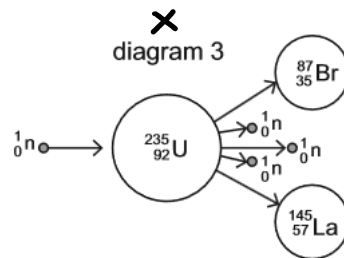
$$\therefore Q:S = 15:20 = 3:4$$

- 6 A uranium-235 nucleus can undergo fission to produce two smaller nuclei.

Which of the diagrams, if any, could represent this process?



Check
LWM for
all
diagrams



Initial + U = Final +
daughter
nuclei

- A none of them
B 1 only
C 2 only
D 3 only
E 1 and 2 only
F 1 and 3 only
G 2 and 3 only
H 1, 2 and 3

- 7 The mean age of the twenty members of a running club is exactly 28.

The mean age increases by exactly 2 years when two new members join.

What is the mean age of the two new members?

- A 20 years
B 22 years
C 30 years
D 40 years
☒ E 50 years
F 52 years

$$\begin{aligned} 28 \times 20 &= 560 = \text{sum before} \\ \text{New sum} &= 22 \times 20 = 660 \\ 660 - 560 &= 100 \\ \frac{100}{2} &= 50 \end{aligned}$$

- 8 A circuit consists of a 5.0Ω resistor and a variable resistor connected in series with a 24 V battery. The variable resistor has a minimum resistance of 3.0Ω and a maximum resistance of 15Ω . The battery and the connecting wires have negligible resistance.

What is the maximum power dissipated in the 5.0Ω resistor?

- A 7.2 W
B 18 W
C 27 W
☒ D 45 W
E 72 W
F 75 W

→ Max power is when $R_{\text{var}} = 3 \Omega$

$$V_{\text{res}} = \frac{5}{8} \times 24 = 15 \text{ V}$$

$$P = \frac{V^2}{R} = \frac{225}{5} = 45 \text{ W}$$

- 9 A medical scanner is bought for £15 000.

The value of the scanner depreciates by 20% every year.

By how much has the scanner reduced in value after 2 years?

- A £600
B £3000
☒ C £5400
D £6000
E £9000
F £9600
G £12 000

$$\begin{aligned} 15000 \times 0.8 &= 12000 \\ 12000 \times 0.8 &= 9600 \\ 15000 - 9600 &= 5400 \end{aligned}$$

- 10 The total power P radiated by a star is given by:

$$P = kR^2T^4$$

where R is the radius of the star, T is its surface temperature and k is a constant.

The power currently radiated by the Sun is $4.0 \times 10^{26} \text{ W}$. Towards the end of the Sun's life its radius will increase by a factor of a hundred and its surface temperature will decrease by a factor of two.

What will be the power radiated by the Sun when these changes have occurred?

- A $2.5 \times 10^{27} \text{ W}$
B $1.0 \times 10^{28} \text{ W}$
C $2.0 \times 10^{28} \text{ W}$
☒ D $2.5 \times 10^{29} \text{ W}$
E $1.0 \times 10^{30} \text{ W}$
F $2.0 \times 10^{30} \text{ W}$
G $2.5 \times 10^{33} \text{ W}$
H $1.0 \times 10^{34} \text{ W}$

$$\begin{aligned} R \uparrow \text{ by } \times 100 &\leftarrow P \uparrow \text{ by } \times 10000 \\ T \downarrow \text{ by } \times 2 &\therefore P \downarrow \text{ by } \times 16 \\ \therefore \text{overall, } P \uparrow \text{ by } &\times \frac{10000}{16} \\ 4 \times 10^{26} \times \frac{10000}{16} &= 0.25 \times 10^{30} \end{aligned}$$

- 11 The point A is 4 km due East of the point B.

The bearing of the point C from A is 330° and the bearing of C from B is 060°

Find the distance BC.

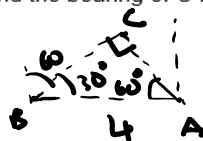
A 2 km

☒ B $2\sqrt{3}$ km

C 4 km

D $2\sqrt{5}$ km

E $4\sqrt{2}$ km



$$\cos 30^\circ = \frac{BC}{4} = \frac{\sqrt{3}}{2}$$

$$BC = 2\sqrt{3}$$

- 12 A transverse wave travelling through a medium has a frequency of 5.0 Hz, a wavelength of 4.0 cm and an amplitude of 3.0 cm.

What is the total distance travelled by a particle of the medium in one minute?

A 900 cm

B 1200 cm

C 1800 cm

D 2400 cm

☒ E 3600 cm

F 4800 cm

In one cycle, particle travels

12 cm ($\uparrow \downarrow \uparrow$)

\therefore In 5 cycles, 60 cm

In 1 min, $60 \times 60 = \underline{3600 \text{ cm}}$

- 13 The quantities x and y are positive.

x is inversely proportional to the square root of y .

When $x = 8$, $y = 9$.

What is the value of y when $x = 6$?

A $\frac{3}{2}$

B 2

C $\frac{81}{16}$

D $\frac{27}{14}$

E 12

☒ F 16

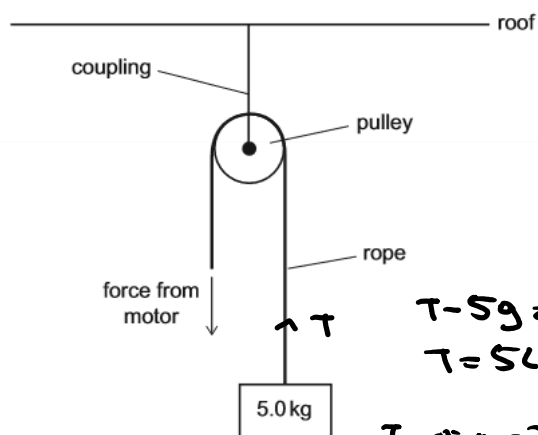
$$x = \frac{k}{\sqrt{y}}$$

$$8 = \frac{k}{3} \therefore k = 24$$

$$6 = \frac{24}{\sqrt{y}} \therefore \sqrt{y} = 4$$

$$y = 16$$

- 14 A motor is used to lift a mass of 5.0 kg using a pulley system as shown in the diagram. The pulley is secured to the roof using a coupling.



$$T - 5g = 5a$$

$$T = 5(10) + 5(0.8) = 54 \text{ N}$$

$$T_{\text{coupling}} = 2 \times 54 = 108 \text{ N}$$

The motor needs to cause the mass to accelerate upwards at 0.80 m s^{-2} .

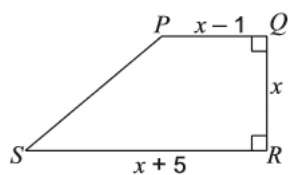
What is the minimum tension force that the coupling must be able to withstand without breaking?

(The gravitational field strength g is 10 N kg^{-1} . The pulley system is frictionless and has negligible mass. The rope has negligible mass and is inextensible.)

- A 4.0 N
- B 8.0 N
- C 46 N
- D 50 N
- E 54 N
- F 92 N
- G 104 N
- H 108 N**

- 15 In a trapezium $PQRS$, the parallel sides are PQ and RS .

$PQ = (x-1)$ cm, $RS = (x+5)$ cm and the vertical height $QR = x$ cm.



[diagram not to scale]

The area of the trapezium is 120 cm^2 .

What is the length of RS ?

A 9 cm

B 10 cm

C 11 cm

D 12 cm

E 15 cm

F 17 cm

$$\frac{1}{2} (x+5 + x-1)(x) = 120$$

$$\frac{1}{2} (2x+4)(x) = 120$$

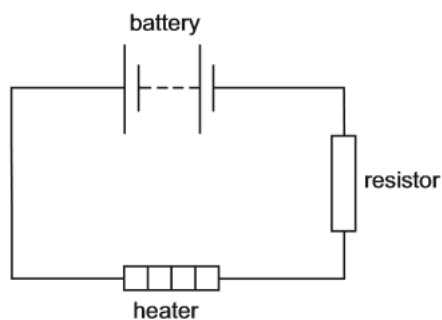
$$x^2 + 2x - 120 = 0$$

$$(x+12)(x-10) = 0$$

$$x \text{ is a length } \therefore x = 10$$

$$RS = 15 \text{ cm}$$

- 16 A heater is connected in series with a resistor and a 6.0 V battery in the circuit shown.



The total resistance of the circuit is 15Ω . In 3.0 minutes, 180 J of electrical energy is transferred into other forms in the heater.

How much charge flows through the heater in the 3.0 minutes and what is the voltage across the heater?

	charge / C	voltage / V
A	1.2	150
B	1.2	216
C	7.5	0.041
D	7.5	24
E	72	0.40
F	72	2.5
G	450	0.40
H	450	2.5

$$I = \frac{6}{15} = 0.4 \text{ A}$$

$$0.4^2 R = \frac{180 \text{ J}}{180 \text{ s}} = 1 \text{ W}$$

$$R = \frac{1}{0.16}$$

$$Q = 0.4 \times 180 = 72 \text{ C}$$

$$V = \frac{1}{0.16} \times 0.4 = \frac{1}{0.4} = 2.5$$

- 17 Make b the subject of the formula:

$$a = \frac{b^2 + 2}{3b^2 - 1}$$

A $b = \pm \sqrt{\left(\frac{a+2}{3a+1}\right)}$

B $b = \pm \sqrt{\left(\frac{a+2}{3a-1}\right)}$

C $b = \pm \sqrt{\left(\frac{2-a}{3a+1}\right)}$

D $b = \pm \sqrt{\left(\frac{2-a}{3a-1}\right)}$

E $b = \pm \sqrt{\left(\frac{3}{3a+1}\right)}$

F $b = \pm \sqrt{\left(\frac{3}{3a-1}\right)}$

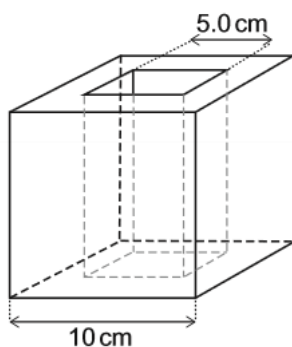
$$3ab^2 - a = b^2 + 2$$

$$b^2(3a-1) = a+2$$

$$b^2 = \frac{a+2}{3a-1}$$

$$b = \pm \sqrt{\frac{a+2}{3a-1}}$$

- 18 A cubic block has a hole through it with a square cross-section. The dimensions are shown on the diagram. The weight of the block is 30 N.



What is the density of the material from which the block is made?

(The gravitational field strength g is 10 N kg^{-1} .)

A 0.30 g cm^{-3}

B 0.40 g cm^{-3}

C 0.60 g cm^{-3}

D 1.2 g cm^{-3}

E 3.0 g cm^{-3}

F 4.0 g cm^{-3}

G 6.0 g cm^{-3}

H 12 g cm^{-3}

$$V = 10^3 - 10(5^2)$$

$$= 1000 - 250 = 750 \text{ cm}^3$$

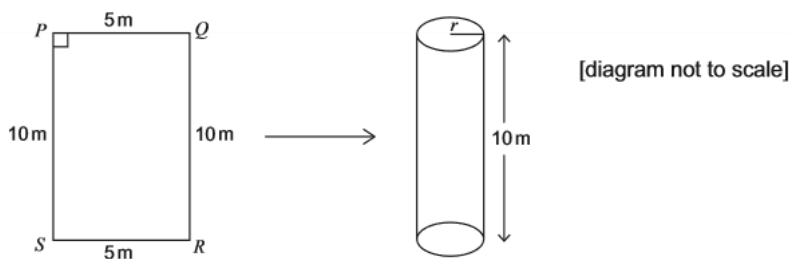
$$\rho = \frac{30/g}{750} = \frac{3}{750} = \frac{1}{250}$$

$$= 0.004 \text{ kg/m}^3$$

- 19 A thin rectangular sheet of metal 10 m by 5 m is made into an open ended cylinder by joining the edges PS and QR .

The height of the cylinder is 10 m.

What is the volume, in cubic metres, enclosed by this cylinder?



A $\frac{5}{2\pi}$

B $\frac{25}{4\pi}$

C $\frac{125}{2\pi}$

D 62.5π

E $\frac{125}{\pi}$

F 250π

$$\text{Volume} = \pi r^2 h$$

$$2\pi r = 5 \therefore r = \frac{5}{2\pi}$$

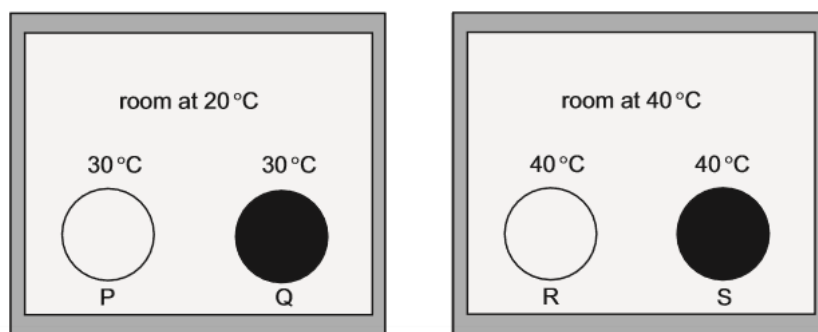
$$V = \pi \left(\frac{25}{4\pi^2} \right) (10) = \frac{125}{2\pi}$$

20 The diagram shows four solid steel balls P, Q, R and S which are of identical size.

Balls P and R have shiny surfaces. Balls Q and S have dull surfaces.

Balls P and Q are in a room at 20 °C. Balls R and S are in a room at 40 °C.

The temperature of each ball at a given moment in time is shown on the diagram.



PQ

Which two balls lose thermal energy by convection, and which ball emits thermal radiation at the greatest rate? S

	<i>lose thermal energy by convection</i>	<i>greatest rate of emission of thermal radiation</i>
A	P and Q	P
B	P and Q	Q
C	P and Q	R
D	P and Q	S
E	R and S	P
F	R and S	Q
G	R and S	R
H	R and S	S

- 21 Which one of the following is a simplification of $4 + \frac{4-x^2}{x^2-2x}$?

☒ A $3 - \frac{2}{x}$

B $3 + \frac{2}{x}$

C $4 - \frac{2}{x}$

D $4 + \frac{2}{x}$

E $5 - \frac{2}{x}$

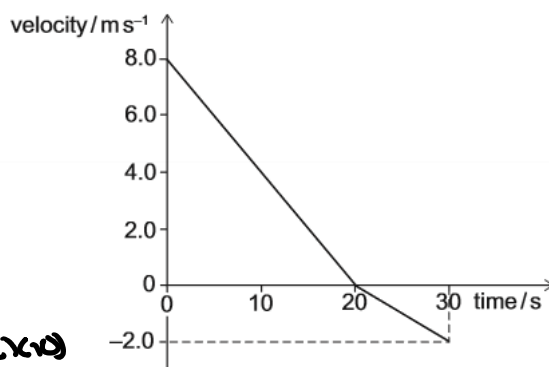
F $5 + \frac{2}{x}$

$$\Rightarrow 4 + \frac{-(2+x)(2-x)}{x(x-2)}$$

$$= 4 - \frac{(2+x)}{x}$$

$$= \frac{4x - 2 - x}{x} = 3 - \frac{2}{x}$$

- 22 The diagram shows the velocity-time graph for an object travelling in a straight line over a period of 30 s.



$$\frac{1}{2}(8 \times 20) + \frac{1}{2}(2 \times 10) = 80 + 10 = 90$$

$$80 - 10 = 70$$

What total distance did the object travel in the 30 s, how far from its starting position was it at the end of the 30 s, and what was its average speed over the 30 s?

$$\frac{90}{30} = 3$$

	total distance travelled / m	distance from starting position / m	average speed / ms ⁻¹
A	90	70	3.0
B	90	70	5.0
C	90	90	3.0
D	90	90	5.0
E	180	140	5.0
F	180	140	6.0
G	180	180	5.0
H	180	180	6.0

- 23 During summer activities week 120 students each chose one activity from swimming, archery, and tennis.

46 of the students were girls.

36 of the students chose tennis, and $\frac{2}{3}$ of these were boys; 25 girls chose swimming, and 27 students chose archery.

A boy is picked at random. What is the probability that he chose swimming?

A $\frac{3}{20}$

B $\frac{9}{37}$

C $\frac{4}{15}$

☒ D $\frac{16}{37}$

E $\frac{32}{57}$

	<u>S</u>	<u>A</u>	<u>T</u>	<u>Total</u>
<u>B</u>	32		24	74
<u>G</u>	25		12	46
<u>Total</u>	57	27	36	120

$$\frac{32}{74} = \frac{16}{37}$$

- 24 Bronze is a mixture of tin and copper.

A particular sample of bronze contains 10% tin by volume. (In other words, 10% of the total volume of the sample is tin and 90% of it is copper.)

What percentage of the mass of the sample is tin?

(Density of tin = Y and density of copper = X .)

A $\frac{X}{9X-Y} \times 100$

B $\frac{X}{9Y-X} \times 100$

C $\frac{Y}{9X-Y} \times 100$

D $\frac{Y}{9Y-X} \times 100$

E $\frac{X}{9X+Y} \times 100$

F $\frac{X}{9Y+X} \times 100$

G $\frac{Y}{9X+Y} \times 100$

H $\frac{Y}{9Y+X} \times 100$

$$M_{\text{Bronze}} = 0.1Y + 0.9X$$

$$\% \text{ Tin} = \frac{0.1Y}{0.1Y + 0.9X}$$

- 25 Which one of the following expressions is equivalent to $\frac{9^{2n+1} \times 3^{4-3n}}{27^{2-n}}$?

A 3^9

B 3^{-2n}

C 3^{2-2n}

D 3^{4n}

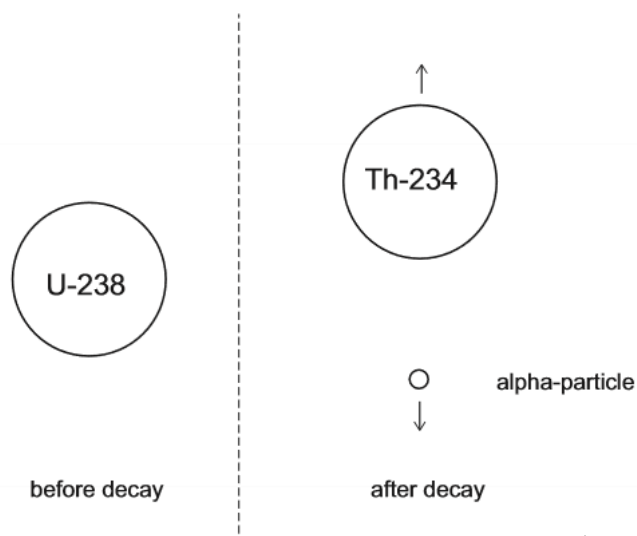
E 3^{6n-2}

F 3^6

$$= \frac{(3^2)^{2n+1} \times 3^{4-3n}}{(3^3)^{2-n}}$$

$$= \frac{3^{4n+2+4-3n}}{3^{6-3n}} = 3^{n+6-6+3n} = 3^{4n}$$

- 26 When a stationary uranium-238 nucleus decays by alpha emission it forms a nucleus of thorium-234. The total kinetic energy produced by the decay is E .



What is the kinetic energy of the alpha particle?

A $\frac{4E}{238}$

B $\frac{4E}{234}$

C $\frac{E}{2}$

☒ D $\frac{234E}{238}$

E E

α has $\frac{4}{234} \times$ the mass

$\therefore \frac{234}{4} \times$ velocity (same)

This means it has $\frac{234}{4} \times$ the KE

of Th.

$$KE_{\alpha} + KE_{Th} = E$$

$$\frac{234}{4} KE_{Th} = E \therefore KE_{Th} = \frac{4E}{238}$$

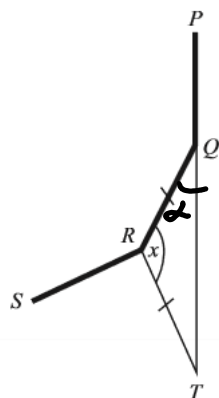
$$\therefore KE_{\alpha} = \frac{234E}{238}$$

- 27 In the diagram below, $PQRS$ is part of a regular polygon.

The polygon has n sides.

The side PQ is extended to T such that PQT is a straight line.

The length of RQ is the same as the length of RT .



[diagram not to scale]

$$\alpha = \frac{180-x}{2} = 90 - \frac{x}{2}$$

$$\text{Int. angle} = \frac{(n-2)(180)}{n}$$

$$\therefore \cancel{180} - \frac{360}{n} + 90 - \frac{x}{2} = 180$$

Find an equation for n in terms of x , where x is the size of angle $\angle QRT$ in degrees.

A $n = \frac{180}{x-90}$

B $n = \frac{180-x}{720}$

C $n = \frac{360-x}{90}$

D $n = \frac{360}{180-x}$

E $n = \frac{720}{180-x}$

F $n = \frac{720}{360-x}$

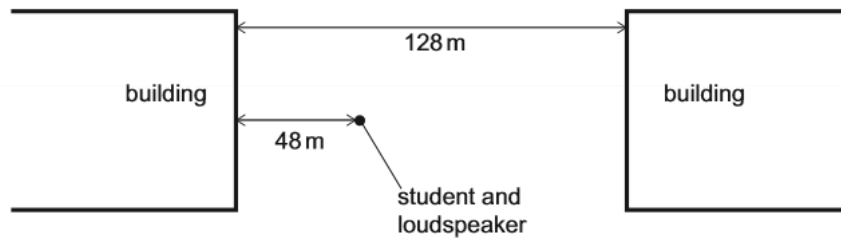
G $n = \frac{360}{360-x}$

$$90 = \frac{x}{2} + \frac{360}{n}$$

$$\frac{180-x}{2} = \frac{360}{n}$$

$$n = \frac{720}{180-x}$$

- 28 A student carries out an experiment to measure the speed of sound. A loudspeaker that emits sound in all directions is placed between two buildings that are 128 m apart as shown. The student and loudspeaker are 48 m from one of the buildings.



The loudspeaker is connected to a signal generator that causes it to emit regular clicks. The student notices that each click results in two echoes, one from each building. The rate at which the clicks are produced is gradually increased from zero until each echo coincides with a new click being emitted by the loudspeaker.

What is the frequency of emission of clicks when this happens?

(The speed of sound in air = 320 m s^{-1} .)

- A 2.0 Hz
- B 2.5 Hz
- C 3.3 Hz
- D 4.0 Hz
- E 5.3 Hz
- F 6.7 Hz
- ☒ G 10 Hz

$$T \text{ for 1st echo} = \frac{96}{320} \text{ s}$$

$$T \text{ for 2nd echo} = \frac{160}{320} \text{ s}$$

$$\frac{(2 \times 128 - 48)}{320}$$

$$T \text{ time between echoes} = \frac{64}{320} \text{ s}$$

$$\therefore \text{Frequency} = \frac{2 \times \frac{320}{64}}{\text{echo}} = \underline{\underline{10 \text{ Hz}}}$$

PART B Advanced Mathematics and Advanced Physics

NOTE: questions in this part that are not covered by the ESAT content specification are indicated by a cross through the question number.

- 29 When $x = 2$ is substituted in the expression $x^3 + px^2 + qx + p^2$ the result is 0.

When $x = 1$ is substituted into the same expression, the result is -3.5 .

Find all possible value(s) of p .

A $p = -1 \pm \frac{\sqrt{6}}{3}$

B $p = 1$ or $p = -3$

☒ C $p = 1$

D $p = 1 \pm \sqrt{7}$

E there are no values for p

$$8 + 4p + 2q + p^2 = 0$$

$$(1 + p + 2 + p^2 = -3.5) \times 2$$

$$2 + 2p + 2q + 2p^2 = -7$$

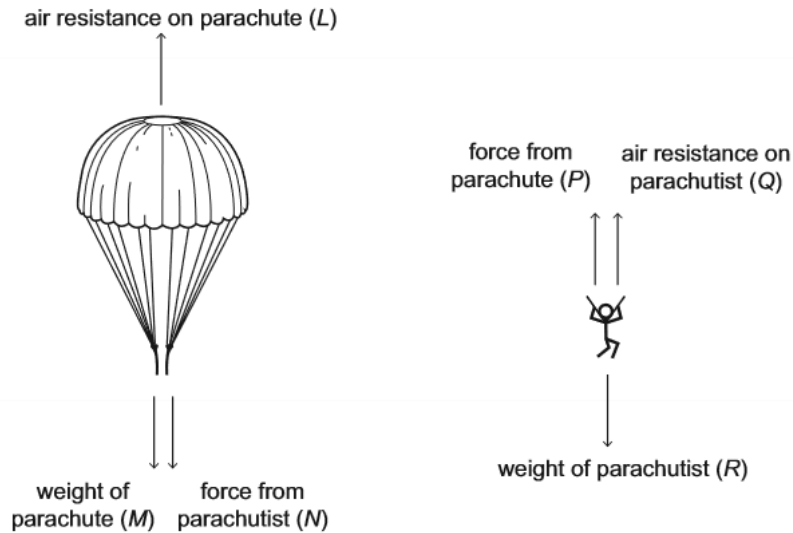
$$\therefore 6 + 2p + 0 - p^2 = 7$$

$$p^2 - 2p + 1 = 0$$

$$(p-1)^2 = 0$$

- ✗ A parachutist is falling at terminal speed with his parachute open. The diagrams show, separately, the vertical forces acting on the parachute and the vertical forces acting on the parachutist.

The letters L , M , N , P , Q and R represent the magnitude of each force as indicated.



Consider the following equations:

Equation 1: $L = M + N$

Equation 2: $R = P + Q$

Equation 3: $L = Q$

Equation 4: $N = P$ ✓

Equation 5: $M + R = L + Q$

Which of these equations, if any, is/are the direct result of the application of Newton's Third Law to this situation?

A none of them

B 3 only

C 4 only

D 5 only

E 1 and 2 only

F 3 and 4 only

G 1, 2 and 5 only

H 1, 2, 3, 4 and 5

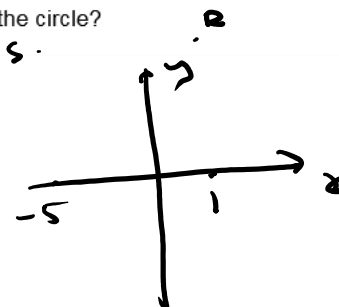
- 31 A square PQRS is drawn above the x -axis with the side PQ on the x -axis.

P is the point $(-5, 0)$ and Q is the point $(1, 0)$.

A circle is drawn inside the square with diameter equal in length to the side of the square.

Which one of the following is an equation of the circle?

- A $x^2 + y^2 - 4x + 6y + 4 = 0$
- B $x^2 + y^2 - 4x + 6y + 9 = 0$
- C** $x^2 + y^2 + 4x - 6y + 4 = 0$
- D $x^2 + y^2 + 4x - 6y + 9 = 0$
- E $x^2 + y^2 - 6x - 4y + 9 = 0$
- F $x^2 + y^2 - 6x + 4y + 4 = 0$
- G $x^2 + y^2 + 6x - 4y + 4 = 0$
- H $x^2 + y^2 + 6x + 4y + 9 = 0$



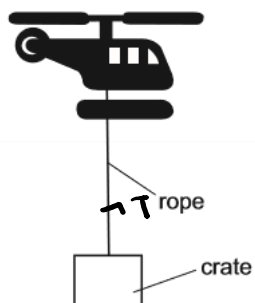
Centre = $(-2, 3)$

Radius = 3

$$\therefore (x+2)^2 + (y-3)^2 = 9$$

$$x^2 + 4x + y^2 - 6y + 4 = 0$$

- ✗ A crate has a total mass of 800 kg, including its contents. A helicopter of mass 4200 kg is carrying the crate using a light inextensible rope as shown:



$$T - 800g = 800(2)$$

$$T = 800(12)$$

$$= 9600 \text{ N}$$

The helicopter and crate are accelerating upwards at 2.0 m s^{-2} .

What is the tension in the rope?

(The gravitational field strength g is 10 N kg^{-1} ; air resistance can be ignored.)

*(Only need to consider
FBD of
crate)*

- A 6400 N
- B 8000 N
- C 9600 N**
- D 18000 N
- E 40000 N
- F 42000 N
- G 50000 N
- H 60000 N

- 33 The first term of a convergent geometric series is 8.

The fifth term is 2.

The sixth term is real and positive.

What is the sum to infinity of this series?

(The sum to infinity of a convergent geometric series is given by $\frac{a}{1-r}$, where a is the first term and r is the common ratio.)

A $8(1+\sqrt{2})$

B $8(1-\sqrt{2})$

☒ C $8(2+\sqrt{2})$

D $8(2-\sqrt{2})$

E 16

F $\frac{8\sqrt[5]{4}}{\sqrt[5]{4}-1}$

G $\frac{8\sqrt[5]{4}}{\sqrt[5]{4}+1}$

$$a = 8$$

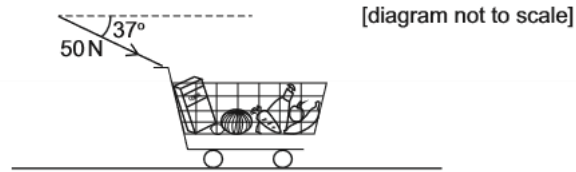
$$ar^4 = 2 \Rightarrow r^4 = \frac{2}{8} = \frac{1}{4}$$

$$r = \pm \frac{1}{\sqrt{2}}$$

6th term is true $\therefore r = \frac{1}{\sqrt{2}}$

$$S_{\infty} = \frac{8}{1 - \frac{1}{\sqrt{2}}} = \frac{8\sqrt{2}}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = 16 + 8\sqrt{2}$$

- ✗ A shopper pushes a supermarket trolley a distance of 15 m in a straight line across a level, horizontal surface. The shopper applies a constant force of 50 N at an angle of 37° below the horizontal. The total weight of the trolley and its contents is 350 N.



What is the magnitude of the total vertical force that the surface exerts on the trolley and how much work is done by the pushing force?

(You may use the approximations $\sin 37^\circ = 0.60$; $\cos 37^\circ = 0.80$.)

	vertical force / N	work done / J
A	380	600
B	380	750
C	390	450
D	390	750
E	400	450
F	400	600

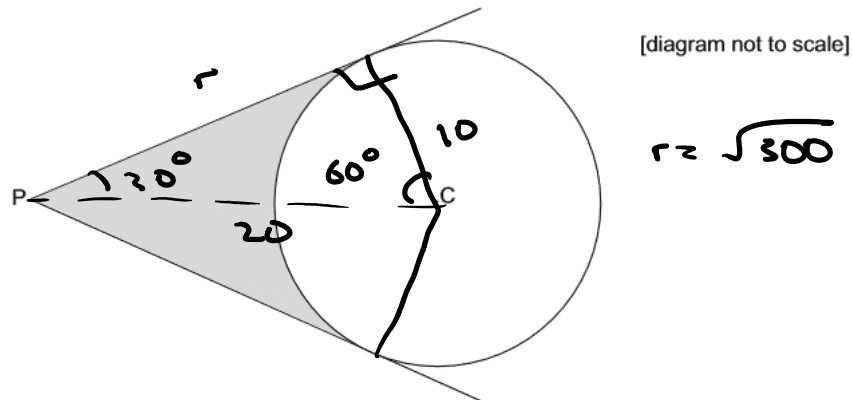
$$F_v = 350 + 50 \sin 37^\circ$$

$$= 350 + 30 = 380$$

$$W_d = 50 \cos 37^\circ \times 15$$

$$= 600$$

- 35 Tangents are drawn from a point P to a circle of radius 10 cm.
The centre of the circle is C and the distance PC is 20 cm.



Which one of the following is an expression for the shaded area in square centimetres?

☒ A $\frac{100}{3}(3\sqrt{3} - \pi)$

B $\frac{100}{3}(3\sqrt{5} - \pi)$

C $\frac{50}{3}(6\sqrt{3} - \pi)$

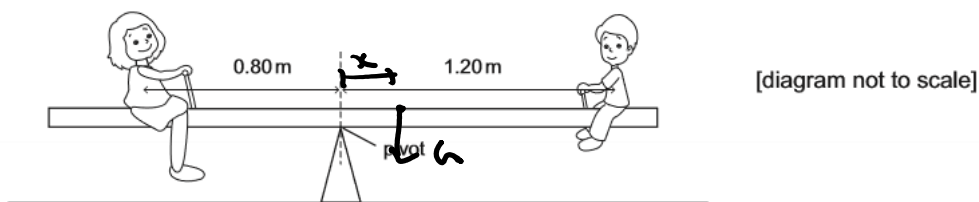
D $\frac{50}{3}(6\sqrt{5} - \pi)$

E $\frac{50}{3}(\sqrt{3} - 2\pi)$

F $\frac{50}{3}(2\pi - \sqrt{3})$

$$\begin{aligned} \text{Shaded} &= 2 \times \frac{1}{2} \times \sqrt{300} \times 10 \\ &\quad - 2 \times \frac{60}{360} \times \pi \times 10^2 \\ &= 100\sqrt{3} - \frac{100}{3}\pi \end{aligned}$$

- ✗ A plank of non-uniform density which has a mass of 15 kg is used to make a see-saw. A pivot is placed under the centre of the plank as shown on the diagram.



A boy of mass 35 kg sits at one end of the plank with his centre of gravity 1.20 m from the pivot. The see-saw balances when a woman of mass 60 kg sits on the plank on the other side of the pivot. Her centre of gravity is 0.80 m from the pivot.

Where is the centre of gravity of the plank and what is the magnitude of the force between the pivot and the plank?

(The gravitational field strength g is 10 N kg^{-1} .)

	distance from pivot	force / N
A	0.40 m on left of pivot	100
B	0.40 m on left of pivot	1100
C	at the pivot	100
D	at the pivot	1100
E	0.20 m on right of pivot	100
F	0.20 m on right of pivot	1100
G	0.40 m on right of pivot	100
H	0.40 m on right of pivot	1100

$$35(1.2) + 15(x) = 60(0.8)$$

$$x = \frac{48 - 42}{15} = 0.4 \text{ m}$$

Right

$$R_{\text{tot}} = 350 + 600 + 150$$

$$= 1100 \text{ N}$$

- 37 Given that $7\cos\theta - 3\tan\theta \sin\theta = 1$, which one of the following is true?

A $\cos\theta = -\frac{3}{5}$ or $-\frac{1}{2}$

B $\cos\theta = -\frac{3}{5}$ or $\frac{1}{2}$

C $\cos\theta = \frac{3}{5}$ or $\frac{1}{2}$

D $\cos\theta = \frac{3}{5}$ or $-\frac{1}{2}$

$$7\cos^2\theta - 3\sin^2\theta = \cos\theta$$

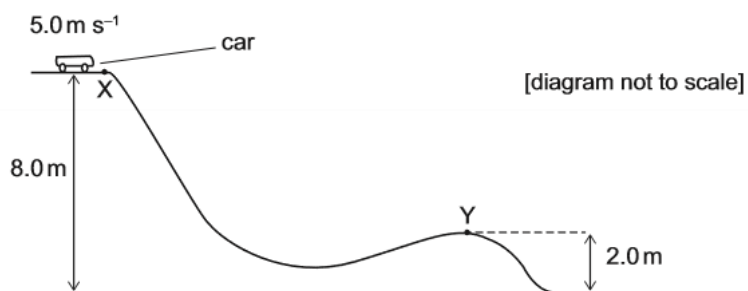
$$10\cos^2\theta - \cos\theta - 3 = 0$$

$$10v^2 - v - 3 = 0$$

$$10v^2 + 5v - 6v - 3 = 0$$

$$5v(2v+1) - 3(2v+1) = 0$$

- ✗ A car of mass 200 kg on a fairground ride travels at a speed of 5.0 m s^{-1} at point X. The car is allowed to move down a sloping section of track without any energy input. The heights above the ground of points X and Y are shown. When the car reaches point Y its speed is 9.0 m s^{-1} .



How much energy is transferred in overcoming resistive forces as the car travels from X to Y?

(The gravitational field strength g is 10 N kg^{-1} .)

A 3900 J

☒ B 6400 J

C 7900 J

D 10400 J

E 11200 J

$$\Delta PE = 200(10)(6) = 12000$$

$$\Delta KE = \frac{1}{2}(200)(9^2 - 5^2) = 5600$$

- 39 The complete set of values of a for which the equation $3x^2 = (a+2)x - 3$ has two real distinct roots is

A no values of a

B $-4\sqrt{2} < a < 4\sqrt{2}$

C $a < -4\sqrt{2}, a > 4\sqrt{2}$

D $-4 < a < 8$

E $a < -4, a > 8$

F $-8 < a < 4$

☒ G $a < -8, a > 4$

H all values of a

$$3x^2 - (a+2)x + 3 = 0$$

$$b^2 - 4ac > 0$$

$$(a+2)^2 - 36 > 0$$

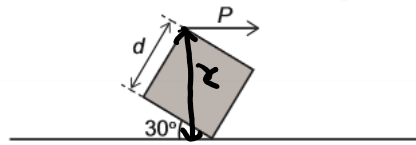
$$(a+2)^2 - 36 = 0$$

$$a = -8 \text{ or } a = 4$$

$$\begin{array}{l} \text{Graph of } y = (a+2)^2 - 36 \\ \text{The parabola opens upwards and crosses the x-axis at } a = -8 \text{ and } a = 4. \\ \text{For two real distinct roots, } y > 0, \text{ we need } a < -8 \text{ or } a > 4. \end{array}$$

- ~~X~~ The diagram shows a uniform, solid, heavy cube with side d . The cube rests with one of its edges in contact with a table that is perfectly level. A horizontal force P acts on another edge of the cube, and the cube is stationary.

[diagram not to scale]



Below are four statements about the forces on the cube.

- 1 It is possible that there is no frictional force between the cube and the table. ~~X~~
- 2 There must be a frictional force acting to the left between the cube and the table. ✓
- 3 There must be a frictional force acting to the right between the cube and the table. ~~X~~
- 4 Force P has a clockwise moment about the edge in contact with the table equal to $P \times d$. ~~X~~
 $\times > d$

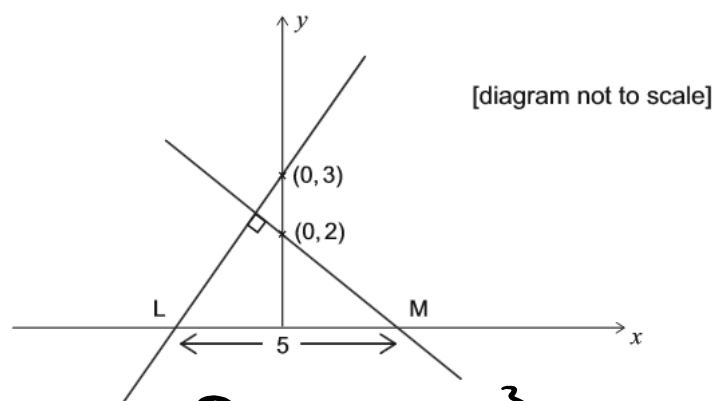
Which of the statements is/are correct?

- ~~A~~ 1 only
 (B) 2 only
 C 3 only
 D 1 and 4 only
 E 2 and 4 only
 F 3 and 4 only

- 41 The straight line with equation $y = mx + 3$, where $m > 0$, $m \neq 1$, is perpendicular to the line with equation $y = px + 2$

The lines cut the x -axis at the points L and M respectively. The length of LM is 5 units.

What is the value of $m + p$ given that $m > 1$?



A $-\frac{8}{3}$

B $-\frac{13}{6}$

C $-\frac{5}{6}$

D $\frac{5}{6}$

E $\frac{13}{6}$

F $\frac{8}{3}$

① $y = 0 \Rightarrow x = -\frac{3}{m} = L$

② $y = 0 \Rightarrow x = -\frac{2}{p} = M$

$p = -\frac{1}{m} \Rightarrow x = 2m$

$2m - (-\frac{3}{m}) = 5$

$2m^2 - 5m + 3 = 0$

$(2m - 3)(m - 1) = 0$

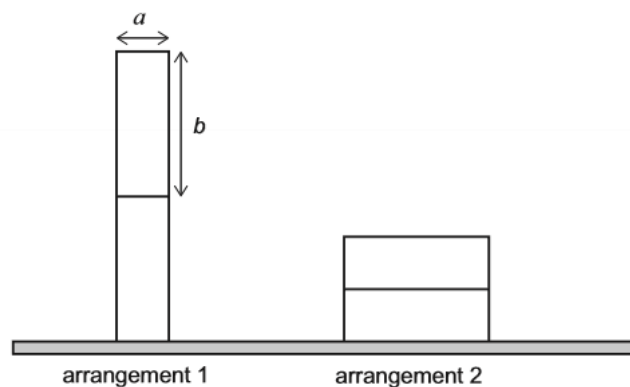
$m > 1 \therefore m = \frac{3}{2}$

$p = -\frac{2}{3}$

$p + m = 5/6$

~~✗~~ The diagram shows two identical blocks, each of mass m , in two different arrangements.

[diagram not to scale]



Which expression gives:

$$\left[\begin{array}{c} \text{gravitational potential energy} \\ \text{of arrangement 2} \end{array} \right] - \left[\begin{array}{c} \text{gravitational potential energy} \\ \text{of arrangement 1} \end{array} \right] ?$$

(g is the gravitational field strength.)

(A) $2mg(a - b)$

B $2mg(b - a)$

C $-mg(b + a)$

D $mg(a + b)$

E $\frac{3}{2}mg(a - b)$

F $\frac{1}{2}mg(a - b)$

① = $mg(\frac{b}{2}) + mg(\frac{3b}{2})$

= $2mgb$

② = $2mga$

- 43 $f(x) = x^3 - a^2x$ where a is a positive constant.

Find the complete set of values of x for which $f(x)$ is an increasing function.

A $x \leq -a, x \geq a$

B $-a \leq x \leq a$

C $x \leq -a, 0 \leq x \leq a$

D $-a \leq x \leq 0, x \geq a$

E $x \leq -\frac{a}{3}, x \geq \frac{a}{3}$

F $-\frac{a}{3} \leq x \leq \frac{a}{3}$

☒ G $x \leq -\frac{a}{\sqrt{3}}, x \geq \frac{a}{\sqrt{3}}$

H $-\frac{a}{\sqrt{3}} \leq x \leq \frac{a}{\sqrt{3}}$

$$\frac{dy}{dx} > 0 \text{ for all } x$$

$$f'(x) = 3x^2 - a^2$$

$$3x^2 - a^2 > 0$$

$$3x^2 - a^2 = 0$$

$$x = \pm \frac{a}{\sqrt{3}}$$



$$\therefore x < -\frac{a}{\sqrt{3}} \text{ or } x > \frac{a}{\sqrt{3}}$$

- ☒ An object is fired vertically upwards from the ground at time $t = 0$ s in still air at a speed of 8.0 m s^{-1} .

On the way up, what is the height of the object above the ground when it has a speed of 2.0 m s^{-1} , and at what time does it reach this height on the way down?

(The gravitational field strength g is 10 N kg^{-1} . Air resistance can be ignored.)

	height / m	time / s
A	2.4	0.60
B	2.4	0.64
C	2.4	1.0
D	2.4	2.0
E	3.0	0.60
F	3.0	0.64
<input checked="" type="radio"/> G	3.0	1.0
H	3.0	2.0

$$v^2 = u^2 + 2as$$

$$u = 8.0 \text{ m s}^{-1}$$

$$s = 3 \text{ m}$$

$$3 = 8t - 5t^2$$

$$5t^2 - 8t + 3 = 0$$

$$(5t - 3)(t - 1) = 0$$

- 45 The curve $y = x^2$ is translated by the vector $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ and then reflected in the line $y = -1$

Which one of the following is an equation of the resulting curve?

A $y = -3 - (x - 4)^2$

B $y = -3 + (x + 4)^2$

C $y = 3 - (x + 4)^2$

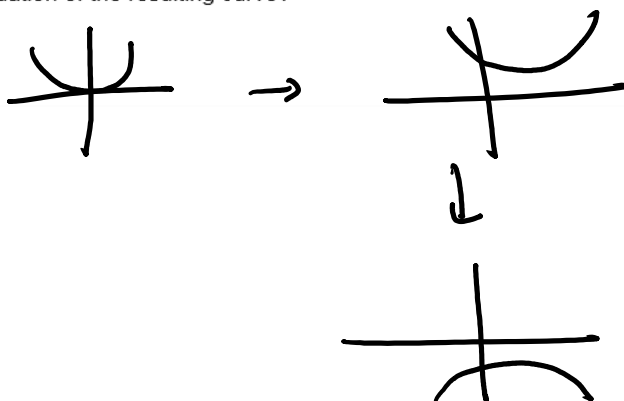
D $y = 3 + (x - 4)^2$

☒ E $y = -5 - (x - 4)^2$

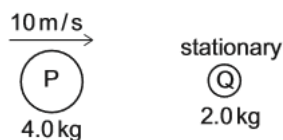
F $y = -5 + (x + 4)^2$

G $y = 5 - (x + 4)^2$

H $y = 5 + (x - 4)^2$



- 46 The diagram shows a ball P, of mass 4.0 kg, moving to the right at 10 m s^{-1} directly towards a stationary ball Q, of mass 2.0 kg.



The balls collide but do not join together. Immediately after the collision ball Q moves at 10 m s^{-1} to the right.

What is the velocity of ball P immediately after the collision, and how much kinetic energy in total is lost during the collision?

	velocity of ball P after collision	kinetic energy lost during collision / J
A	0	50
B	0	150
C	10 m s^{-1} to the left	50
D	10 m s^{-1} to the left	150
<input checked="" type="radio"/> E	5.0 m s^{-1} to the <u>right</u>	50
F	5.0 m s^{-1} to the <u>right</u>	150

$$4(10) + 2(0) = 4x + 2y$$

$$40 = 4x + 2y$$

$$y = 10$$

$$40 = 4x + 20 \Rightarrow x = 5 \rightarrow$$

$$KE_{\text{int}} = \frac{1}{2}(4)(100) = 200$$

$$KE_{\text{final}} = \frac{1}{2}(4)(25) + \frac{1}{2}(2)(100) \\ = 50 + 100 = 150$$

47 The complete set of values of x for which $2x^4 - 9x^2 + 4 > 0$ is

A $x < \frac{1}{2}, x > 4$

B $\frac{1}{2} < x < 4$

C $x < -2, -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}, x > 2$

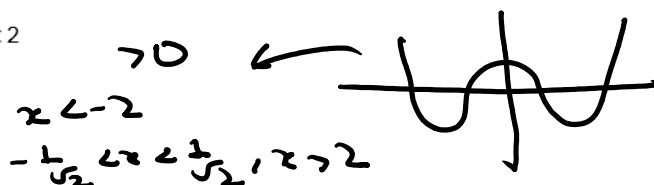
D $-2 < x < -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} < x < 2$

E $-2 < x < 2$

$$2x^4 - 9x^2 + 4 > 0$$

$$(2x^2 - 1)(x^2 - 4) = 0$$

$$x = \pm \frac{1}{\sqrt{2}} \text{ or } x = \pm 2$$



~~X~~ A point object of mass 2.0 kg is at rest on a level, horizontal surface. The coefficient of friction between the object and the surface is 0.25.

Two horizontal forces at right-angles to each other, with magnitudes 9.0 N and 12.0 N, are applied simultaneously to the object.

What is the magnitude of the acceleration of the object as it begins to move?

(The gravitational field strength g is 10 N kg^{-1} .)

A 5.0 ms^{-2}

B 7.25 ms^{-2}

C 7.5 ms^{-2}

D 8.0 ms^{-2}

E 10 ms^{-2}

F 10.5 ms^{-2}

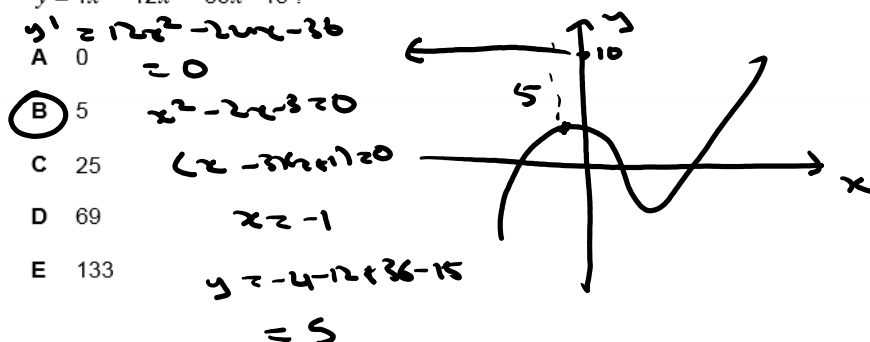


$$\text{Friction} = 2 \times 10 \times 0.25 = 5 \text{ N}$$

$$a = \frac{15 - 5}{2} = 5 \text{ ms}^{-2} //$$

- 49 A cursor starts at the point (0, 10) and moves parallel to the x -axis in the negative direction.

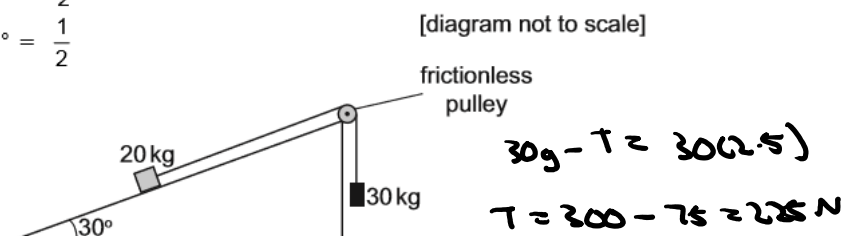
What is the minimum distance parallel to the y -axis between the cursor and the graph of $y = 4x^3 - 12x^2 - 36x - 15$?



- ~~50~~ An object of mass 20 kg is pulled up a rough plane inclined at 30° to the horizontal by a light, inextensible cable attached via a frictionless pulley to a freely-falling 30 kg mass. The acceleration of the object along the plane is 2.5 m s^{-2} .

$$\cos 30^\circ = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\sin 30^\circ = \cos 60^\circ = \frac{1}{2}$$



What is the frictional force between the object and the plane?

(Air resistance and the mass of the pulley can be ignored. The gravitational field strength g is 10 N kg^{-1} .)

- A 25 N

- B 50 N

- C 75 N

- D 100 N

- E 150 N

- F 175 N

- G 250 N

$$225 - F_f - 200 \sin 30 = 20(2.5)$$

$$F_f = 225 - 200 \sin 30 - 50$$

$$= 75 \text{ N}$$

- 51 What is the area enclosed by the line $x=7$ and the curve $x=3(y-1)^2+4$?

☒ A 4

B 8

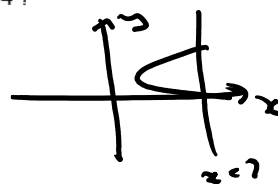
C 10

D 11

E 14

F 20

$$\begin{aligned}
 3(y-1)^2 + 4 &= 7 \\
 (y-1)^2 &= 1 \Rightarrow y = 0 \text{ or } 2 \\
 \text{Area} &= \int_0^2 7 - 3(y-1)^2 - 4 \, dy \\
 &= \left[-y^3 + 3y^2 \right]_0^2 = -8 + 12 = \underline{4}
 \end{aligned}$$



- ☒ A spacecraft of initial total mass 4000 kg is travelling relative to the Earth at a constant speed of 7425 m s^{-1} .

It ejects some fuel backwards in a sudden burst at a speed relative to the spacecraft of 1425 m s^{-1} . As a result of this, the speed of the spacecraft immediately after the fuel is ejected increases to 7500 m s^{-1} .

What is the mass of fuel ejected?

A 22 kg

B 34 kg

C 40 kg

D 50 kg

☒ E 200 kg

F 210 kg

$$4000(7425) = -M_f(1500) + 4000(7500)$$

$$M_f = \frac{4000(75)}{1500} = \frac{4000}{20} = 200$$

- 53 A curve has equation $y = 3x^4 - 4x^3 - 12x^2 + 20$

What is the complete set of values of the constant k for which the equation

$$3x^4 - 4x^3 - 12x^2 + 20 = k$$

has exactly four distinct real roots?

A no values of k

B $-12 < k < 15$

☒ C $15 < k < 20$

D $k > 20$

E $7 < k < 20$

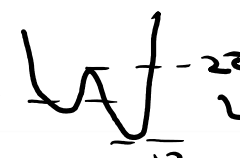
F all values of k

$$y' = 12x^3 - 12x^2 - 24x = 0$$

$$x(x^2 - x - 2) = 0$$

$$x(x - 2)(x + 1) = 0 \rightarrow x = 0, -1 \text{ or } 2$$

$$y = 20, 15, 12$$

1 <  4 roots for k between 15 and 20

- ☒ An object of weight 40 N hangs from the end of a light inextensible string of length 0.35 m, which is attached to the ceiling. A constant horizontal force of 30 N is applied to the object, causing it to move to a new equilibrium position with the string no longer vertical.

By how much has the gravitational potential energy of the object increased as a result of its change of position?

A 2.1 J

☒ B 2.8 J

C 3.5 J

D 4.2 J

E 4.9 J

F 5.6 J



$$T' \cos \theta = 40$$

$$T' \sin \theta = 30$$

$$\tan \theta = \frac{3}{4}$$



$$\begin{aligned} \Delta WPE &= 40(0.35 - 0.35 \cos \theta) \\ &= 40(0.35 - 0.28) = 2.8 \end{aligned}$$