ENGAA 2023

UNIVERSITY OF CAMBRIDGE

ENGINEERING ADMISSIONS ASSESSMENT **D564/11**

2023 **60 minutes**

SECTION 1

INSTRUCTIONS TO CANDIDATES

Please read these instructions carefully, but do not open this question paper until you are told that you may do so. This paper is Section 1 of 2.

A separate answer sheet is provided for this paper. Please check you have one. You also require a soft pencil and an eraser.

Please complete the answer sheet with your candidate number, centre number, date of birth, and name.

At the end of 60 minutes, your supervisor will collect this question paper and answer sheet before giving out Section 2.

This paper contains **two** parts, **A** and **B**, and you should attempt **both** parts.

Part A	Mathematics and Physics (20 questions)
Part B	Advanced Mathematics and Advanced Physics (20 questions)

You are **strongly** advised to divide your time equally between the two parts: 30 minutes on **Part A** and 30 minutes on **Part B**. The scores for Part A and Part B are reported separately.

This paper contains 40 multiple-choice questions. There are no penalties for incorrect responses, only marks for correct answers, so you should attempt **all** 40 questions. Each question is worth one mark.

For each question, choose the **one** option you consider correct and record your choice on the separate answer sheet. If you make a mistake, erase thoroughly and try again.

You **must** complete the answer sheet within the time limit.

You can use the question paper for rough working, but **no extra paper** is allowed. Only your responses on the answer sheet will be marked.

Dictionaries and calculators are NOT permitted.

Please wait to be told you may begin before turning this page.

This question paper consists of 31 printed pages and 5 blank pages.

 PV4

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PART A Mathematics and Physics

1 The surface area of a solid sphere of radius R is equal to the total surface area of 10 solid closed cylinders of radius r and height $4r$.

Which of the following is an expression for R in terms of r?

(The surface area of a sphere of radius R is $4\pi R^2$.)

A $R = 5r$

B $R = 12r$

C $R = 2\sqrt{5}\,r$

D $R = \frac{1}{2}\sqrt{10}\,r$

E $R = \sqrt{10}\,r$

F $R = \frac{3}{2}\sqrt{10}\,r$

G $R = \sqrt{15}\,r$

$SA_{sphere} = 4\pi R^2$

$SA_{cylinder} = 2\pi rh + 2\pi r^2$

$= 2\pi r(4r) + 2\pi r^2$

$= 10\pi r^2$

$4\pi R^2 = 10(10\pi r^2)$

$R^2 = 25r^2 \therefore R = 5r$

2 A spaceship of mass 10 000 kg is moving at 2.0 m s^{-1} relative to a space station.

The spaceship is captured by a robotic arm attached to the space station and brought to rest by a force of 1000 N.

How far will the spaceship move in its initial direction relative to the space station while the force is being applied?

(Assume that the acceleration of the space station is negligible.)

A 0.050 m

B 0.10 m

C 0.20 m

D 5.0 m

E 10 m

F 20 m

$Wd = KE$ lost

$1000 \times d = \frac{1}{2}(10000)(4)$

$\cancel{1000}d = 20\cancel{000}$

3 Which of the following is a correct rearrangement of

$$y = p - \frac{q-r}{s-x}$$

to make x the subject?

$$y - p = -\frac{q-r}{s-x}$$

$$p - y = \frac{q-r}{s-x}$$

$$s - x = \frac{q-r}{p-y}$$

$$x = s - \frac{q-r}{p-y}$$

A $x = s - \dfrac{q-r}{p+y}$

B $x = \dfrac{q-r}{p+y} - s$

C $x = s - \dfrac{q-r}{p-y}$

D $x = \dfrac{q-r}{p-y} - s$

E $x = s - \dfrac{q-r}{y-p}$

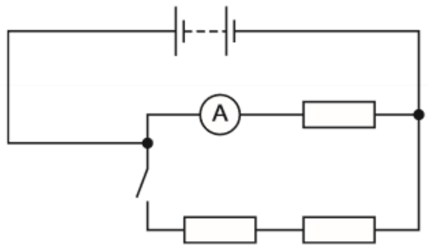
F $x = \dfrac{q-r}{y-p} - s$

4 A circuit is set up as shown. All three resistors are identical.

When the switch is open, the reading on the ammeter is 1.0 A and the power transferred from the battery is 1.0 W.

$I^2R = 1 \therefore R = 1\Omega$

$V = 1V$



The switch is now closed.

What is the new reading on the ammeter and what is the new power transferred from the battery?

$R_{tot} = \frac{1\times2}{1+2} = \frac{2}{3}$

$P_{tot} = \frac{V^2}{R} = 1.5\,W$

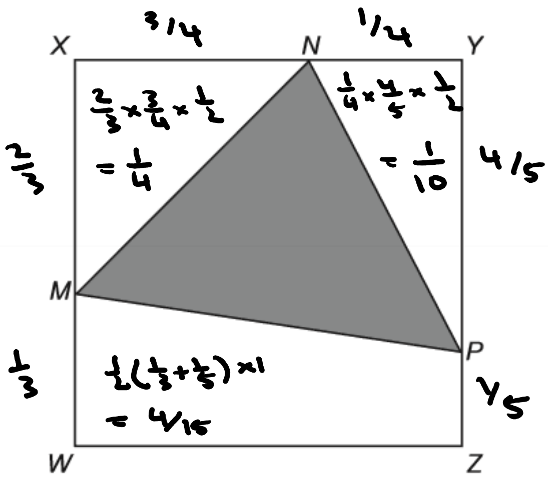
$I_A = \frac{2}{3} \times I_{tot}$

$= \frac{2}{3} \times \frac{3}{2}$

$= \underline{\underline{1A}}$

	ammeter reading / A	*power transferred* / W
A	0.67	0.67
B	0.67	1.3
C	0.67	1.5
D	0.67	2.0
E	1.0	1.0
F	1.0	1.5
G	1.0	2.0
H	1.0	3.0

5



[diagram not to scale]

WXYZ is a square of side length 1.

$WM : MX = 1 : 2$

$XN : NY = 3 : 1$

$YP : PZ = 4 : 1$

What is the area of triangle *MNP*?

A $\frac{1}{3}$

B $\frac{2}{5}$

C $\frac{9}{20}$

D $\frac{1}{30}$

E $\frac{19}{60}$

F $\frac{23}{60}$

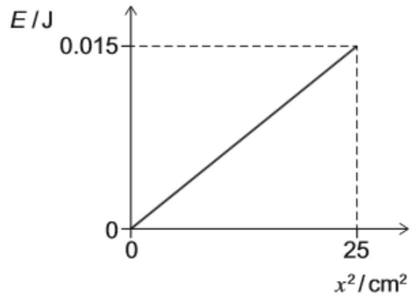
$A_{square} = 1$

$1 - \frac{1}{4} - \frac{1}{10} - \frac{4}{15}$

$= 1 - \left(\frac{15+6+16}{60}\right) = \frac{23}{60}$

6 A spring is initially unstretched. A force F is used to stretch the spring. The extension x and the energy E stored in the stretched spring are measured for different values of F.

The graph shows how the energy E, in J, varies with the extension squared, x^2, in cm^2.



What is the magnitude of F when the spring stores 0.015 J of energy?

A 0.30 N

B 0.60 N

C 1.2 N

D 1.5 N

E 2.4 N

F 3.0 N

G 30 N

H 60 N

$\frac{1}{2}Fx = 0.015$

$F(0.05) = 0.03$

$F = 0.\underline{6\ N}$

7 Given that

$$\frac{27^{2(x-2)}}{9^{(2x-3)}} = (81)^{\frac{3}{2}}$$

what is the value of x?

$$\frac{3^{3(2x-4)}}{3^{2(2x-3)}} = (3^4)^{3/2}$$

A 0

B 2.5

C 3

D 6

E 7.5

F 9

G 10.5

H 12

$$\frac{3^{6x-12}}{3^{4x-6}} = 3^6$$

$$3^{6x-12-4x+6} = 3^6$$

$$6 = 2x - 6 \therefore x = \underline{6}$$

8 A solid, cylindrical metal bar has a uniform cross-sectional area of 12 cm^2 and a volume of 180 cm^3.

The bar rests on a horizontal surface on one of its circular faces.

The pressure on the surface due to the bar is 0.45 N cm^{-2}.

What is the density of the metal, in g cm^{-3}?

(gravitational field strength = 10 N kg^{-1})

$F = P \times A$

$= 0.45\,N/cm^2 \times 12\,cm^2$

$= 5.4\,N$

A 2.5 g cm^{-3}

B 3.0 g cm^{-3}

C 3.75 g cm^{-3}

D 7.5 g cm^{-3}

E 15 g cm^{-3}

F 33 g cm^{-3}

Mass = 0.54 kg

$\rho = \frac{\underline{540g}}{180cm^3} = 3g/cm^3$

9 Last year, the salary of the coach of a football club was 80% of the salary of the star player.

At the start of the new year, the coach received a 15% increase in salary and the star player received a 38% increase in salary.

What percentage of the star player's new salary is the coach's new salary?

A 46%

B 57%

C $61\frac{3}{5}$%

D $66\frac{2}{3}$%

E 77%

F $83\frac{1}{3}$%

LY → Coach salary = x

Player = x/0.8

TY → Coach salary = 1.15x

Player = $\frac{1.38x}{0.8}$

$1.15x \div \frac{1.38x}{0.8} = \frac{0.92}{1.38} = \frac{2}{3}$

10 Two samples of pure radioactive isotopes X and Y decay with half-lives of 2 days and 3 days, respectively.

Both X and Y decay in a single step into different stable isotopes.

Initially the number of atoms of X is twice the number of atoms of Y.

After how many days are the expected numbers of atoms of X and Y equal to each other?

A The expected numbers of atoms of X and Y are never equal.

B 2 days

C 3 days

D 4 days

E 6 days

F 12 days

T	X	Y
0	2	1
1	—	—
2	1	—
3	—	1/2
4	1/2	—
5	—	—
6	1/4	1/4

11 An athlete's training session consists of several complete repetitions of a three-part programme:

1. Walk 100 m at an average speed of $6\,\text{km h}^{-1}$
2. Jog 200 m at an average speed of $10\,\text{km h}^{-1}$
3. Run 100 m at an average speed of $20\,\text{km h}^{-1}$

What is the athlete's average speed for the complete training session, in km h^{-1}?

A 7.2

B 9.6

C 11.5

D 12

E 14.4

Total distance = 0.4 km

$$\text{Total time} = \frac{0.1}{6} + \frac{0.2}{10} + \frac{0.1}{20}$$

$$= \frac{1 + 1.2 + 0.3}{60} = \frac{2.5}{60} = \frac{5}{120} \text{ hrs}$$

$$\text{Speed} = 0.4 \div \frac{5}{120} = \frac{48}{5}$$

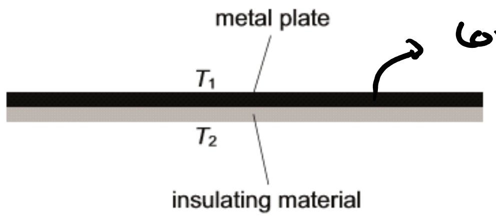
12 A large, flat, metal plate is coated on one side with a layer of thermally insulating material of the same thickness *a* as the metal plate.

The uninsulated top surface of the metal plate is maintained at a constant temperature T_1.

The bottom surface of the insulating material is maintained at a constant, lower temperature T_2.

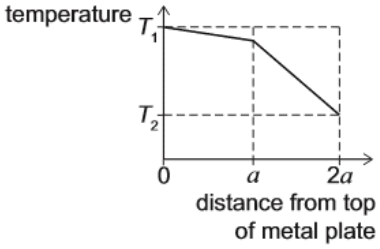
The system is in equilibrium.

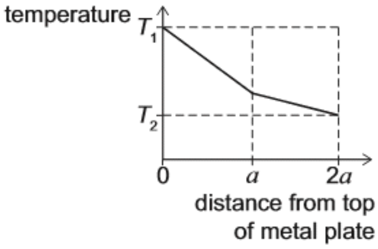
The diagram shows this arrangement.



conducts heat ∴ lower ΔT across thickness

Which graph could show how the temperature varies with distance from the top surface of the metal plate to the bottom surface of the insulating material?

A


B


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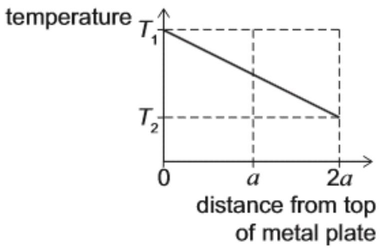
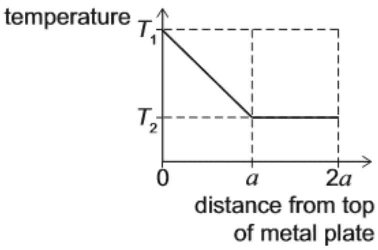
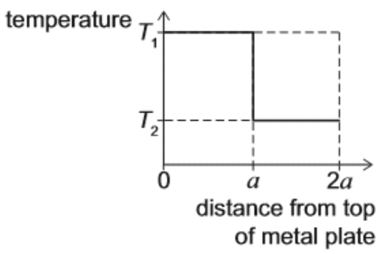


D


E


13 Two objects X and Y are similar.

The surface area of object Y is double the surface area of object X.

The volume of object Y is $7\sqrt{2}\,\text{cm}^3$ more than the volume of object X.

What is the volume of object X, in cm^3?

A $14-7\sqrt{2}$

B $14+7\sqrt{2}$

C $\dfrac{42-7\sqrt{2}}{17}$

D $\dfrac{42+7\sqrt{2}}{17}$

E $\dfrac{7\sqrt{2}}{3}$

F $7\sqrt{2}$

G $4-\sqrt{2}$

H $4+\sqrt{2}$

SA: $Y : X = 2 : 1$

Vol: $Y : X = 2^{3/2} : 1$

$= 2\sqrt{2} : 1$

$V_Y = 2\sqrt{2}V_X = V_X + 7\sqrt{2}$

$V_X = \dfrac{7\sqrt{2}}{2\sqrt{2}-1} \times \dfrac{2\sqrt{2}+1}{2\sqrt{2}+1}$

$= \dfrac{28+7\sqrt{2}}{7} = 4+\sqrt{2}$

14 The voltage output of a power station is stepped up using a transformer before the power is transmitted to a distant town. The primary coil of this transformer has 300 turns and the secondary coil has 1500 turns.

In the town, a step-down transformer reduces the voltage supplied by the transmission cables to 33 000 V for distribution within the town. The step-down transformer supplies a current of 1500 A.

The current in the transmission cables is 450 A and both transformers are ideal and 100% efficient.

What is the voltage output of the power station?

(Assume that the resistance of the transmission cables is negligible.)

A 1980 V

B 6600 V

C 22 000 V

D 110 000 V

E 550 000 V

$V_c I_c = V_T I_T$

$V_{cable} = \dfrac{33000 \times 150\cancel{0}}{45\cancel{0}} = \dfrac{11000 \times 450}{45}$

$= 110000$

$V_{station} = \dfrac{300}{1500} \times 110\,000 = \underline{22\,000}$

15 The equation

$$\left(\frac{a\times10^4 + 2a\times10^3}{3\times10^{-1}}\right)^2 = 8\times10^9$$

has two solutions for a.

What is the positive difference between these two solutions?

$$\frac{1}{9}\left(\frac{a\times10^4 + 2a\times10^3}{10^{-1}}\right)^2 = 8\times10^9$$

$$\frac{a^2(10^4 + 2\times10^3)^2}{10^{-2}} = 72\times10^9$$

$$a^2 = \frac{72\times10^7}{12000^2} = \frac{72\times10^7}{144\times10^6} = 5$$

$$a = \pm\sqrt{5}$$

A 0

B $2\sqrt{5}$

C $4\sqrt{5}$

D $20\sqrt{5}$

E $40\sqrt{5}$

F $200\sqrt{5}$

16 A transverse wave with an amplitude of 3.0 cm travels along a stretched string. The wave has a frequency of 12 Hz and a wavelength of 0.25 m.

What is the average speed of a particle in the string as the string oscillates during a time of 2.0 s?

Period = $\frac{1}{12}$ s

∴ In 2s, 24 periods

↑↓ ↓↑ = 12cm

Distance = 12×24cm

$\frac{12\times24}{2}$ = 144cm/s

A 36 cm s^{-1}

B 72 cm s^{-1}

C 125 cm s^{-1}

D 144 cm s^{-1}

E 300 cm s^{-1}

17 X and Y are the end-points of a line segment.

Point P has coordinates $(-8, 5)$.

P lies on the line segment XY such that $XP : PY$ is $1 : 2$ and $\overrightarrow{XP} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$

A point Q is such that $\overrightarrow{QY} = \begin{pmatrix} 7 \\ 6 \end{pmatrix}$

What are the coordinates of point Q?

$$\overrightarrow{PY} = \begin{pmatrix} 8 \\ -6 \end{pmatrix}$$

$$Y = \begin{pmatrix} -8 \\ 5 \end{pmatrix} + \begin{pmatrix} 8 \\ -6 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$\overrightarrow{QY} = \begin{pmatrix} 0 \\ -1 \end{pmatrix} - \begin{pmatrix} 7 \\ 6 \end{pmatrix} = \begin{pmatrix} -7 \\ -7 \end{pmatrix}$$

A $(7, 5)$

B $(3, 8)$

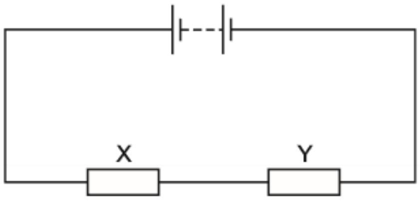
C $(1, -12)$

D $(-3, -10)$

E $(-7, -7)$

F $(-11, -4)$

18 A battery and two resistors X and Y are connected in series.



The power transferred by the battery is 6 W.

The resistance of X is 10 Ω.

The voltage across Y is 4 V.

What is the current in the circuit?

A $\frac{2}{5}$ A

B $\frac{3}{5}$ A

C $\frac{3}{4}$ A

D 1 A

E $\sqrt{\frac{3}{10}}$ A

F $\sqrt{\frac{3}{5}}$ A

$V_{tot} I = 6$

$V_{tot} = 4 + 10I$

$(4+10I)I = 6$

$10I^2 + 4I - 6 = 0$

$5I^2 + 2I - 3 = 0$

$5I^2 + 5I - 3I - 3 = 0$

$5I(I+1) - 3(I+1) = 0$

$I > 0 < I = 3/5$

19 Find the maximum value of

$$2^{\sin x} \times 3^{-\sin x}$$

where $0° \leq x \leq 360°$

$= \frac{2^{\sin x}}{3^{\sin x}} = \left(\frac{2}{3}\right)^{\sin x}$

$= \frac{3}{2}$ when $\sin x = -1$

A $\frac{2}{3}$

B 1

C $\frac{3}{2}$

D 2

E 3

F 6

20 A diver at the bottom of a lake of depth d fills a syringe with an ideal gas and seals the nozzle. The piston remains free to move. The volume of the gas in the syringe at the bottom of the lake is $90\,cm^3$.

As the diver returns to the surface, the temperature of the gas does not change. At the surface of the lake the gas in the syringe is at atmospheric pressure and the volume of the gas is $720\,cm^3$.

What is the volume of the gas in the syringe at a depth $\frac{d}{2}$?

A $160\,cm^3$

B $180\,cm^3$

C $206\,cm^3$

D $225\,cm^3$

E $288\,cm^3$

F $315\,cm^3$

G $360\,cm^3$

H $405\,cm^3$

$P_0V_0 = P_bV_b$

$P_{atm}(720) = (P_d + P_{atm})(90)$

$P_d + P_{atm} = 8P_{atm}$

$P_d = 7P_{atm}$

$P_{d/2} = 3.5\,P_{atm}$

$\cancel{P_{atm}}(720) = (P_{d/2} + P_{atm})(V) = (4.5\cancel{P_{atm}})V$

$V = \frac{720}{4.5} = \underline{\underline{160\,cm^3}}$

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PART B Advanced Mathematics and Advanced Physics

21 $x^2 - x - 6$ is a factor of $x^3 + ax^2 + 2x + b$, where a and b are real constants.

What is the value of $a + b$?

A –39

B –21

C –3

D $-\frac{3}{5}$

E $\frac{3}{5}$

F 3

G 21

H 39

$x^3 + ax^2 + 2x + b = (x^2 - x - 6)(x + c)$

$-c - 6 = 2$

$-c = 8 \therefore c = -8$

$\Rightarrow (x^2 - x - 6)(x - 8)$

$= x^3 - 9x^2 + 2x + 48$

a b

$a + b = \underline{39}$

22 Two pipes contain air at the same temperature and pressure.

A stationary sound wave is formed in the first pipe, which is closed at one end and open at the other end. The lowest frequency of stationary sound wave that can be formed in this pipe is 4000 Hz.

The second pipe has the same length as the first pipe, but is open at both ends.

What is the lowest frequency of stationary sound wave that can be formed in the second pipe?

A 1000 Hz

B 2000 Hz

C 4000 Hz

D 8000 Hz

E 16 000 Hz

1st pipe → length = $\frac{\lambda}{4}$

v is the same

① → $v = f\lambda = f \cdot 4L$

② → $v = f_2\lambda = f_2 \cdot 2L$

$f_2 \cdot 2L = f \cdot 4L \therefore f_2 = \underline{8000\,Hz}$

23 An arithmetic progression has first term a and common difference d.

The sum of the first 9 terms plus the sum of the first 10 terms is equal to the sum of the first 11 terms.

Which of the following is a correct expression for a in terms of d?

A $a = -\frac{13}{4}d$

B $a = -\frac{15}{4}d$

C $a = -\frac{16}{3}d$

D $a = -\frac{19}{3}d$

E $a = -7d$

F $a = -8d$

$S_9 = \frac{9}{2}(2a+8d)$

$S_{10} = 5(2a+9d)$

$S_{11} = \frac{11}{2}(2a+10d)$

$\frac{9}{2}(2a+8d) + 5(2a+9d) = \frac{11}{2}(2a+10d)$

$19a + 81d = 11a + 55d$

$8a = -26d \therefore a = -\frac{13}{4}d$

24 A cannon ball of mass 2.6 kg is fired horizontally from a cannon on the top of a cliff at a speed of $90\,\text{m}\,\text{s}^{-1}$.

The height of the cliff above the horizontal ground below is 45 m.

What is the magnitude of the impulse that acts on the ball between leaving the cannon and reaching the ground?

(gravitational field strength = $10\,\text{N}\,\text{kg}^{-1}$; assume that air resistance is negligible)

A 8.7 N s

B 13 N s

C 52 N s

D 78 N s

E 117 N s

F 234 N s

G 900 N s

Horizontal speed is the same.

Vertically → $v^2 = u^2 + 2as$

$v^2 = 0 + 2(-10)(-45)$

$v^2 = 900 \therefore v = 30$ m/s

Impulse = 0 horizontally

Vertically, $I = m(30-0) = 78$ Ns

25 The first three terms of a convergent geometric progression are:

$$2p, \quad p - 3, \quad p - 7$$

What is the sum to infinity of this progression?

$\frac{9-3}{18} = \frac{1}{3}$

A –54

$\frac{p-3}{2p} = \frac{p-7}{p-3}$

B –27

C $-13\frac{1}{2}$

$p^2 - 6p + 9 = 2p^2 - 14p$

D –2

$p^2 - 8p - 9 = 0$

E 2

$(p-9)(p+1) = 0$

F $13\frac{1}{2}$

$p = 9$ or $p = -1$

G 27

For convergent, |common ratio| < 1 $\therefore p = 9$

H 54

$S_\infty = \frac{a}{1-r} = \frac{18}{1 - 1/3} = \underline{\underline{27}}$

26 A child on a sledge is travelling straight down a snow-covered slope at a constant speed of $15\,\text{m}\,\text{s}^{-1}$.

The mass of the child and sledge together is 60 kg.

The angle of the slope to the horizontal is 30°.

What is the rate at which thermal energy is being produced due to friction forces?

(gravitational field strength = $10\,\text{N}\,\text{kg}^{-1}$)

A 450 W

GPE → Thermal energy rate

B 900 W

C 4500 W

$mg\dot{h}$ = Rate of production of TE

D $4500\sqrt{3}$ W

E 6750 W

$60(10)(15\sin 30) = 600 \times 15 \times \frac{1}{2}$

F 9000 W

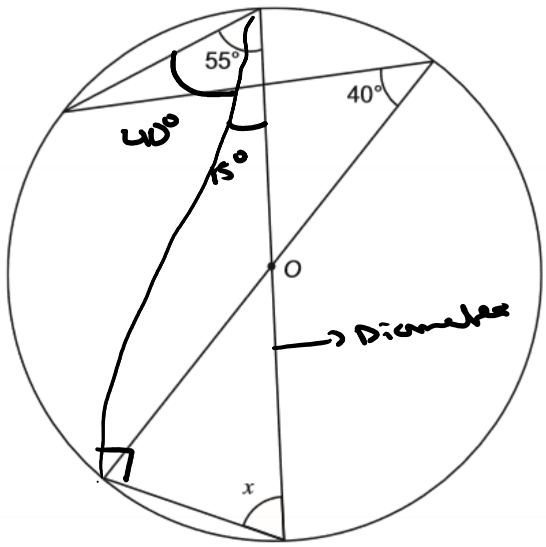
$= 300 \times 15$

G 13 500 W

$= 4500\text{W}$

H $9000\sqrt{3}$ W

27 The diagram shows a circle with centre O.



[diagram not to scale]

$90 + 15 = 105°$

$180 - 105 = \underline{75°}$

What is the value of x?

A 40°

B 50°

C 55°

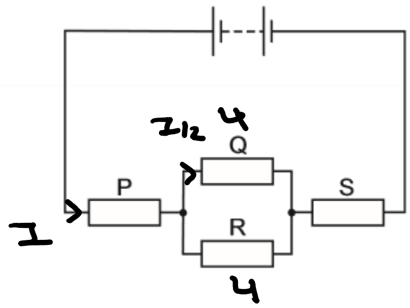
D 70°

E 75°

F 80°

28 The diagram shows a battery with no internal resistance and four identical resistors P, Q, R and S.

Resistor Q dissipates 4.0 W of power.



Power = I^2R

$\therefore P_P = P_S = 4P_Q$

$= 16W$

What is the total power supplied by the battery?

$16 + 16 + 4 + 4 = \underline{40W}$

A 10 W

B 16 W

C 24 W

D 32 W

E 40 W

29 The function

$$f(x) = \sqrt{2}\,x^2 - 6x + 4$$

can be written in the form

$$f(x) = p(x + q)^2 + r$$

where p, q and r are constants.

What is the value of $p(r - q)$?

A 2

B 7

C $3 - \frac{\sqrt{2}}{2}$

D $3 - 7\sqrt{2}$

E $4 - 3\sqrt{2}$

F $4\sqrt{2} - 6$

G $4\sqrt{2} - 12$

H $7\sqrt{2} - 18$

$f(x) = \sqrt{2}(x^2 - 3\sqrt{2}x) + 4$

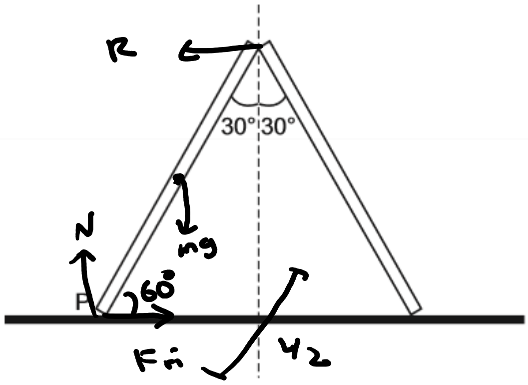
$= \sqrt{2}\left[(x - \frac{3\sqrt{2}}{2})^2 - \frac{9}{2}\right] + 4$

$= \sqrt{2}(x - \frac{3\sqrt{2}}{2})^2 - \frac{9\sqrt{2}}{2} + 4$

$r - q = 4 - \frac{9\sqrt{2}}{2} + \frac{3\sqrt{2}}{2} = 4 - 3\sqrt{2}$

$p(r - q) = 4\sqrt{2} - 6$

30 Two identical, uniform, thin planks, each of mass 40 kg, are propped up against one another in equilibrium on a rough surface as shown. Each plank is at an angle of 30° to the vertical.



[diagram not to scale]

What is the magnitude of the friction force at point P?

(gravitational field strength = 10 N kg^{-1})

A 100 N

B $\frac{200}{\sqrt{3}}$ N

C 200 N

D $200\sqrt{3}$ N

E $\frac{400}{\sqrt{3}}$ N

F 400 N

G $400\sqrt{3}$ N

$\tau_P: \; mg\left(\frac{\cancel{L}}{2}\cos 60\right) = R(\cancel{L}\cos 30)$

$\frac{mg}{4} = \frac{R\sqrt{3}}{2}$

$R = \frac{mg}{2\sqrt{3}} = \frac{40(10)}{2\sqrt{3}}$

$= \frac{200}{\sqrt{3}}$

31 Which of the following is equal to

$$\int_{-(3-\sqrt{5})}^{3-\sqrt{5}} \frac{x^2}{3-\sqrt{5}}\,dx$$

A 0

B $\frac{2}{3}$

C $\frac{8}{3}$

D 4

E $\frac{28-12\sqrt{5}}{3}$

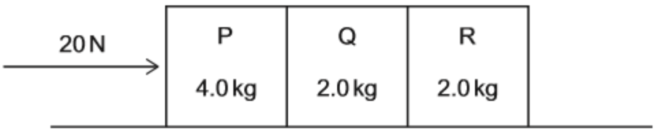
F $28-12\sqrt{5}$

$$= \left[\frac{x^3}{3(3-\sqrt{5})}\right]_{-(3-\sqrt{5})}^{3-\sqrt{5}}$$

$$= \frac{(3-\sqrt{5})^{\cancel{3}\,2}}{3\cancel{(3-\sqrt{5})}} - \frac{\left[-(3-\sqrt{5})\right]^3}{3(3-\sqrt{5})}$$

$$= \frac{9-6\sqrt{5}+5}{3} + \frac{9-6\sqrt{5}+5}{3} = \frac{28-12\sqrt{5}}{3}$$

32 Three blocks P, Q and R are in contact on a horizontal surface as shown.



The mass of P is 4.0 kg and the masses of Q and R are each 2.0 kg.

A horizontal force of 20 N is applied to P so that all three blocks move together.

The friction force between P and the surface is 2.0 N, and the friction forces between Q and R and the surface are each 1.0 N.

What is the magnitude of the force that R exerts on Q?

A 2.0 N

B 5.0 N

C 6.0 N

D 12 N

E 15 N

F 17 N

G 19 N

P: $20 - R_{PQ} - 2 = 4a$

Q: $R_{PQ} - R_{QR} - 1 = 2a$ $\quad + \rightarrow 17 - R_{QR} = 6a$

R: $R_{QR} - 1 = 2a$

$R_{QR} = 2a+1$

$17 - 2a - 1 = 6a$

$a = 2$

$R_{QR} = \underline{\underline{5N}}$

33 A function f is defined by

$$f(x) = \frac{a}{x} + \frac{b}{x^2}$$

where a and b are constants.

It is given that $f'(1) = 2$ and $f''(-1) = -2$

What is the value of $a + b$?

$f'(x) = -\frac{a}{x^2} - \frac{2b}{x^3}$

A -5

$f''(x) = \frac{2a}{x^3} + \frac{6b}{x^4}$

B -4

C $-\frac{7}{5}$

$\times 3 \; (-a - 2b = 2)$ $\quad -2a + 6b = -2$

$-3a - 6b = 6$

D -1

E $\frac{9}{5}$

$-5a = 4$

F 2

$a = -\frac{4}{5}$

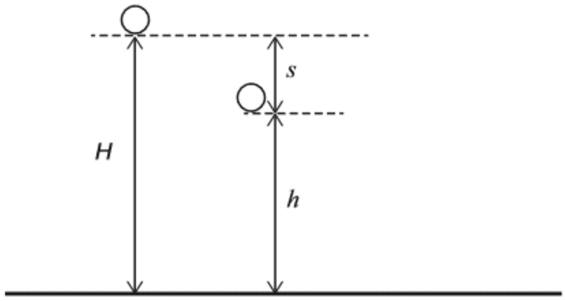
G $\frac{14}{5}$

H 3

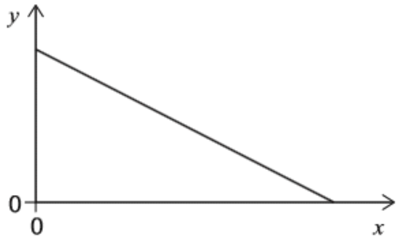
$\frac{4}{5} - 2b = 2$

$b = -\frac{3}{5}$ $\quad \therefore a + b = -\frac{7}{5}$

34 An object is released from rest at a height H above the ground and falls freely in a uniform gravitational field. At time t after being released, it has fallen a distance s and is at a height h above the ground, travelling at speed v.



The graph shows two quantities plotted.



Which of the rows show(s) a possible pair of quantities for the axes?

(Assume that air resistance is negligible.)

	y-axis	x-axis
1	h	t ✗
2	kinetic energy	h ✓
3	gravitational potential energy	s ✓

$h = 0 - 5t^2$ ✗

$mg(H - \underline{h}) = \frac{1}{2}mv^2$

GPE = $mg(h)$

$= mg(H - \underline{s})$

A none of them

B 1 only

C 2 only

D 3 only

E 1 and 2 only

F 1 and 3 only

G 2 and 3 only

H 1, 2 and 3

35 Find the real value of x that satisfies

$\rightarrow \log_3 9$

$$\log_3(x^2 + 3x + 2) = 2 + \log_3(x^2 + 2x)$$

A $\frac{1}{8}$

B $\frac{1}{7}$

C $\frac{1}{6}$

D $\frac{1}{2}$

E 2

F 7

$x^2 + 3x + 2 = 9(x^2 + 2x)$

$8x^2 + 15x - 2 = 0$

$8x^2 + 16x - x - 2 = 0$

$8x(x+2) - 1(x+2) = 0$

$x = \frac{1}{8}$ or $x = -2$

$x > 0 \therefore x = \frac{1}{8}$

36 A 20 V battery of negligible internal resistance is connected in series with a 20 Ω resistor and a cylindrical conductor. The current in the circuit is 0.20 A.

The cylindrical conductor is now removed, melted and re-formed into a new cylinder. After the cylinder has cooled to its original temperature, its length is 4 times greater than that of the original cylinder.

What is the resistance of the new cylinder?

A 5.0 Ω

B 20 Ω

C 320 Ω

D 400 Ω

E 1280 Ω

F 1600 Ω

4 x length

= $\frac{1}{4}$ x Area for same volume

$R = \frac{\rho L}{A} \Rightarrow 16\times$ resistance

$R_{old} = \frac{20}{0.2} - 20 = 80\,\Omega$

$16 \times 80 = 1280\,\Omega$

37 What is the constant term in the simplified binomial expansion of $\left(\frac{2}{x}+\frac{x}{4}\right)^8$?

A $\frac{7}{2}$

B $\frac{35}{8}$

C $\frac{1}{16}$

D $\frac{7}{256}$

E $\frac{35}{1024}$

F $\frac{1}{2048}$

$$\text{Constant} = {}^8C_4 \times \left(\frac{2}{x}\right)^4 \times \left(\frac{x}{4}\right)^4$$

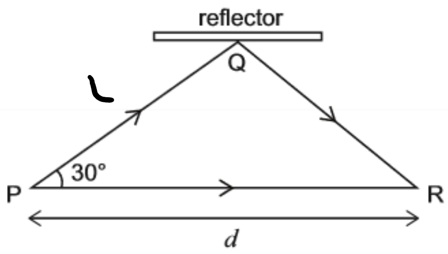
$$= \frac{8!}{4!\cdot 4!} \cdot \frac{\cancel{16}}{\cancel{x^4}} \cdot \frac{\cancel{x^4}}{\cancel{256}\,16}$$

$$= \frac{\cancel{8}\times 7\times 6\times 5}{16\times \cancel{4}\times 3\times \cancel{2}}$$

$$= \frac{210}{48} = \frac{35}{8}$$

38 A sound wave can travel from a source at P to a detector at R directly or by reflecting at Q.

The angle between PR and PQ is 30° and the distance from P to R is d as shown.



There is a phase difference of π radians between the incident and reflected wave at Q.

Waves that reach R via Q are in phase with waves that reach R directly from P.

Which expression gives the greatest wavelength of sound waves for which this is true?

A $2d\left(\frac{2}{\sqrt{3}}-1\right)$

B $d\left(\frac{2}{\sqrt{3}}-1\right)$

C $d(2-\sqrt{3})$

D $\frac{4d}{\sqrt{3}}$

E $\frac{2d}{\sqrt{3}}$

For greatest λ, path difference $= \lambda/2$ (to give phase diff of π)

$L\cos 30 = \frac{d}{2} \therefore L = \frac{d}{\sqrt{3}}$

$2L - d = \frac{\lambda}{2} \therefore \frac{2d}{\sqrt{3}} - d = \frac{\lambda}{2}$

$\lambda = 2d\left(\frac{2}{\sqrt{3}}-1\right)$

39 Given that

$$x^2 + y^2 = 1$$

what is the greatest possible value of $2x + 3y$?

A $\frac{7}{2}$

B 3

C $\frac{5\sqrt{2}}{2}$

D $\sqrt{7}$

E $\sqrt{10}$

F $\frac{13\sqrt{10}}{10}$

G $\sqrt{13}$

H $\frac{12\sqrt{13}}{13}$

$x^2 + y^2 = 1$

$y = \pm\sqrt{1-x^2}$

greatest $\rightarrow y = +\sqrt{1-x^2}$

Let $D = 2x + 3y$

$D = 2x + 3\sqrt{1-x^2}$

$\frac{dD}{dx} = 2 + \frac{\frac{3}{\cancel{2}}(-\cancel{2}x)}{\sqrt{1-x^2}} = 0$

$\therefore 2 = 3x/\sqrt{1-x^2}$

$\rightarrow 4(1-x^2) = 9x^2$

$13x^2 = 4$

$x = \frac{2}{\sqrt{13}}$

$y = \frac{3}{\sqrt{13}}$

$\therefore 2x + 3y = \frac{13}{\sqrt{13}}$

$= \sqrt{13}$

40 Two springs have spring constants of 200 N m^{-1} and 600 N m^{-1}, respectively.

The springs are joined in series, end-to-end, and stretched so that their combined extension is 0.80 m.

What is the total strain energy stored in the springs?

A 48 J

B 96 J

C 112 J

D 120 J

E 240 J

F 256 J

G 512 J

$k_{tot} = \frac{200 \times 600}{200 + 600}$

$= \frac{1200\cancel{00}}{8\cancel{00}} = 150\ \text{N m}^{-1}$

$E = \frac{1}{2}(150)(0.8)^2$

$= 75(0.64) = 48\ \text{J}$

UNIVERSITY OF CAMBRIDGE

ENGINEERING ADMISSIONS ASSESSMENT

D564/12

2023

60 minutes

SECTION 2

INSTRUCTIONS TO CANDIDATES

Please read these instructions carefully, but do not open this question paper until you are told that you may do so. This paper is Section 2 of 2.

A separate answer sheet is provided for this paper. Please check you have one. You also require a soft pencil and an eraser.

Please complete the answer sheet with your candidate number, centre number, date of birth, and name.

This paper contains 20 multiple-choice questions. There are no penalties for incorrect responses, only marks for correct answers, so you should attempt **all** 20 questions. Each question is worth one mark.

For each question, choose the **one** option you consider correct and record your choice on the separate answer sheet. If you make a mistake, erase thoroughly and try again.

You **must** complete the answer sheet within the time limit.

You can use the question paper for rough working, but **no extra paper** is allowed. Only your responses on the answer sheet will be marked.

Dictionaries and calculators are NOT permitted.

Please wait to be told you may begin before turning this page.

This question paper consists of 21 printed pages and 3 blank pages.

PV2

BLANK PAGE

1 A block of weight W slides down a rough plane at a constant speed.

The plane is at an angle of 30° to the horizontal.

The block is now pulled by a force of $3W$ acting parallel to and up the plane. The block has constant acceleration.

Which expression gives the acceleration of the block?

(gravitational field strength = g)

A $2g$

B $\frac{5}{2}g$

C $3g$

D $(3-\sqrt{3})g$

E $\left(3-\frac{1}{\sqrt{3}}\right)g$

F $\left(3-\frac{\sqrt{3}}{2}\right)g$

G $\left(3-\frac{2}{\sqrt{3}}\right)g$

$W\sin 30$ = Friction force (const. speed)

$3W - W\sin 30 - F_{ri} = ma$

$3W - 2W\sin 30 = \frac{W}{g} a$

$\therefore a = 2g$

2 The speed v of an object moving in a straight line is related to time t by the equation

$$v = kt^2$$

where k is a constant.

At $t = 10\,\text{s}$ the speed of the object is $48\,\text{m}\,\text{s}^{-1}$ and the resultant force on the object is 24 N.

What is the mass of the object?

A 0.15 kg

B 0.40 kg

C 1.2 kg

D 2.5 kg

E 6.7 kg

$a = \frac{dv}{dt} = 2kt$

$48 = k(100)$

$\therefore k = 0.48$

$a(10) = 20k$

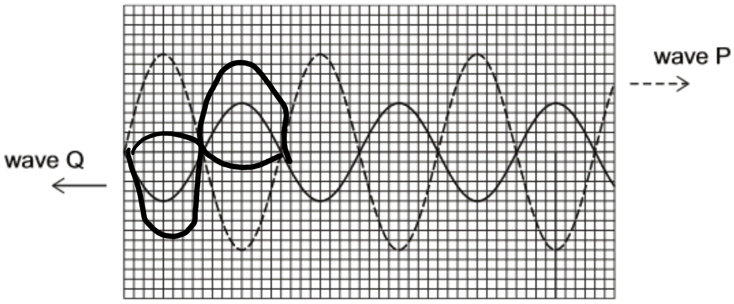
$F = ma$

$24 = m(20k)$

$24 = m(20 \times 0.48) = 9.6m$

$m = \frac{24}{9.6} = \underline{2.5\text{kg}}$

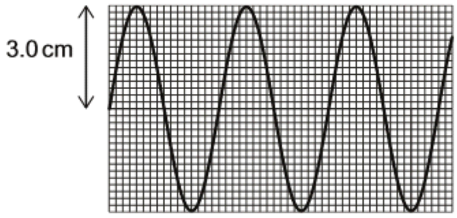
3 Two waves P and Q, which superpose, are shown in the diagram in a particular region at time $t = 0$.



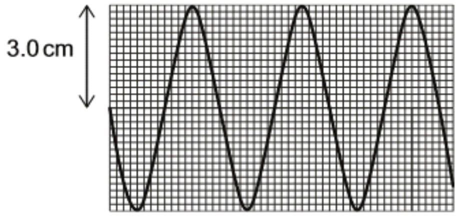
Both waves have period T and are moving in the directions shown by the arrows. Wave P has amplitude 2.0 cm and wave Q has amplitude 1.0 cm.

Which diagram represents the resultant wave formed in the same region by waves P and Q at time $t = \frac{T}{2}$? → ½ a period ∴ diagram reflected!

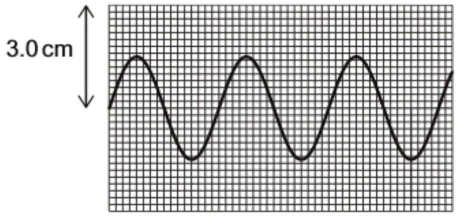
A



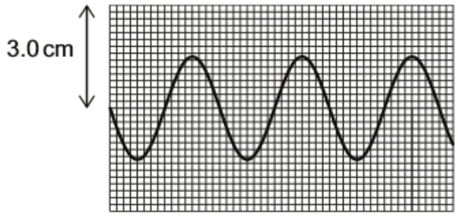
B



C



D



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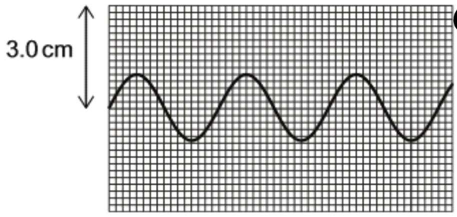
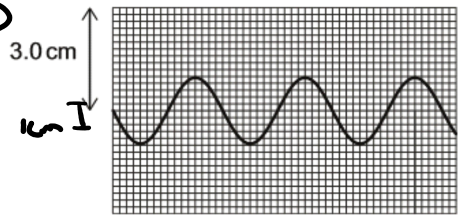
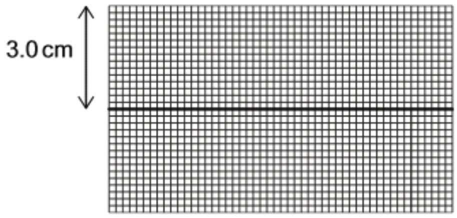


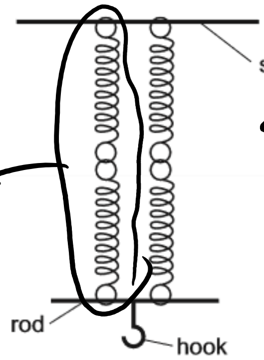
F



G



4 Four identical springs are arranged as shown and suspended from a support.



$\frac{k}{2}$

Series 2 springs:

$k_{tot} = \frac{k \times k}{k+k} = k/2$

k_{tot} parallel $= 2\left(\frac{k}{2}\right)$

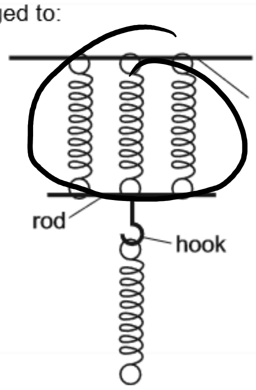
$= k$

The mass of the springs, rod and hook are negligible.

A load of weight 8.4 N is attached to the hook at the lower end of the springs and this causes a total extension of the system of 24 mm.

$k = \frac{F}{x} = \frac{8.4}{0.024}$

The arrangement is then changed to:



$k_{tot} = 3k$

$\frac{3k \times k}{3k + k} = \frac{3k}{4}$

The load of 8.4 N is attached to the bottom of the lower spring.

What is the total extension of the system at equilibrium in the second arrangement?

(The springs obey Hooke's law.)

$k \propto \frac{1}{x}$ (constant F)

$\therefore$ x is $\frac{4}{3}$ of before

$\frac{4}{3} \times 24 = 32$

A 3 mm

B 12 mm

C 16 mm

D 24 mm

E 32 mm

F 48 mm

G 64 mm

5 A student and a child are standing on trolleys X and Y, respectively, which are close to each other but not touching. The trolleys are initially stationary on a straight, horizontal frictionless track. The student is initially holding a ball of mass 5.0 kg.

The total mass of the student, the ball and trolley X is 80 kg.

The total mass of the child and trolley Y is 20 kg.

The student on trolley X throws the ball to the child on trolley Y. The ball travels at a horizontal speed of $12\,\text{m}\,\text{s}^{-1}$ relative to the ground. The child then catches the ball.

What is the speed of separation of the trolleys after the child has caught the ball?

(Assume that air resistance is negligible.)

A $1.6\,\text{m}\,\text{s}^{-1}$

B $2.4\,\text{m}\,\text{s}^{-1}$

C $3.2\,\text{m}\,\text{s}^{-1}$

D $3.8\,\text{m}\,\text{s}^{-1}$

E $24\,\text{m}\,\text{s}^{-1}$

X CoM: $0 = 5(12) + 75(-v)$

$v = -\frac{4}{5}\,\text{ms}^{-1}$ $(\leftarrow)$

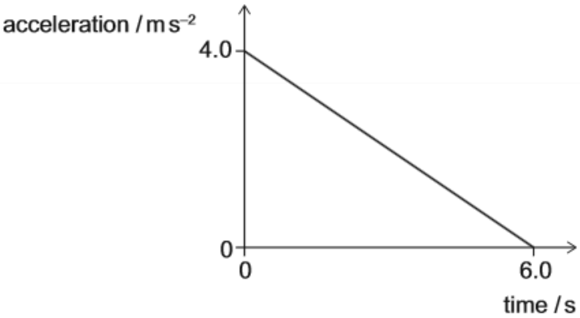
Y CoM: $0 + 12(5) = 25(u)$

$u = \frac{60}{25} = 2.4\,\text{ms}^{-1} \rightarrow$

$2.4 - -0.8 = \underline{\underline{3.2}}$

6 The variation of the acceleration with time of an object moving in a straight line is shown on the graph.

At time = 0 s the velocity of the object is $8.0\,\text{m s}^{-1}$.



What is the maximum velocity of the object between time = 0 s and time = 6 s?

A $5.0\,\text{m s}^{-1}$

B $8.0\,\text{m s}^{-1}$

C $12\,\text{m s}^{-1}$

D $20\,\text{m s}^{-1}$

E $32\,\text{m s}^{-1}$

F $44\,\text{m s}^{-1}$

$a = 4 - \frac{2}{3}t$

V_{max} is when $a = 0 \rightarrow 4 = \frac{2}{3}t$

$t = 6s$

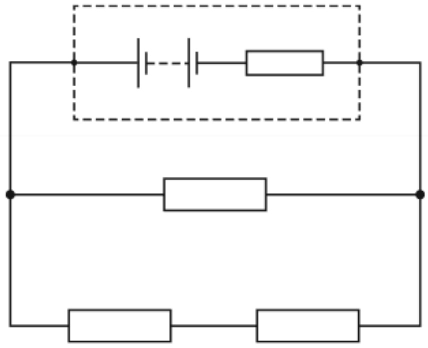
$v = 4t - \frac{2}{3}\frac{t^2}{2} + c = 4t - \frac{t^2}{3} + c$

$v(0) = 8 \therefore c = 8$

$v(6) = 4(6) - \frac{6^2}{3} + 8 = 24 - 12 + 8$

$= 20$

7 The diagram shows a circuit that includes a battery with an emf of 18 V and internal resistance r.



$R_{tot} = \frac{2R \times R}{2R + R} = \frac{2R}{3}$

The three identical resistors in the external circuit each have resistance R.

The terminal potential difference across the battery is 16 V.

Which expression gives R in terms of r?

A $R = \frac{10r}{3}$

B $R = \frac{16r}{3}$

C $R = 6r$

D $R = 12r$

E $R = \frac{27r}{2}$

F $R = 24r$

G $R = \frac{51r}{2}$

$$\frac{\frac{2R}{3}}{r + \frac{2R}{3}} = \frac{16}{18}$$

$$12R = 16r + \frac{32R}{3}$$

$$4R/3 = 16r$$

$$R = \frac{48r}{4} = 12r$$

8 Three identical bar magnets, each of mass m, and two identical trolleys, X and Y, also each of mass m, are arranged with the bar magnets fixed to the trolleys as shown. The trolleys are held at rest a short distance apart on a smooth horizontal track.

The trolleys are released at the same time. They move towards each other and collide.

Find the value of the ratio

$$\frac{\text{kinetic energy of X immediately before collision}}{\text{kinetic energy of Y immediately before collision}}$$

A $\frac{4}{9}$

B $\frac{1}{2}$

C $\frac{2}{3}$

D 1

E $\frac{3}{2}$

F 2

G $\frac{9}{4}$

Same force on both trolleys (N3)

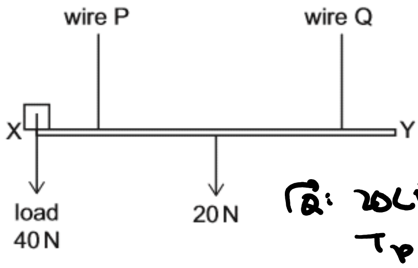
X has $\frac{2}{3}$ the mass $\therefore \frac{3}{2}$ the final velocity (same Force) → WD $= F \times d$ $= \Delta KE$

$\therefore E_{kX} = \frac{3}{2} \times E_{kY}$

$\frac{1}{2}(\frac{2}{3}m)(\frac{3}{2}v)^2$

$\frac{1}{2}mv^2$

9 A uniform rod XY of length 3.0 m has a weight of 20 N. The rod is supported by two light wires, P and Q, as shown. P and Q are attached 0.50 m from ends X and Y, respectively.



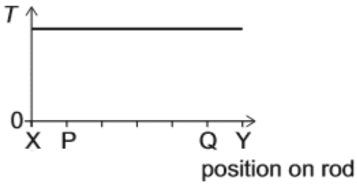
$\circlearrowright Q$: $20(1) + 40(x) = T_p(2)$
$T_p = 10 + 20(x)$
↳ Distance of load from Q

A 40 N load is moved from end X to end Y. The rod remains horizontal at all times.

Which graph shows the variation of the tension T in wire P with the position of the load as it is moved along the rod?

∴ When $x = -0.5$ (@ Y),

$T_p = 0$

A


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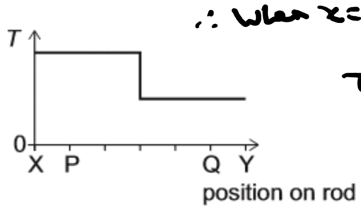

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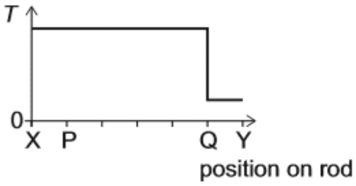

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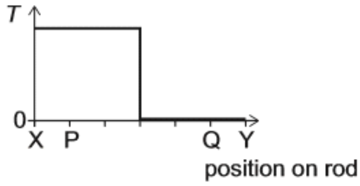

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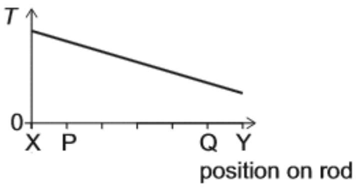

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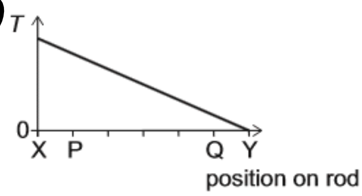
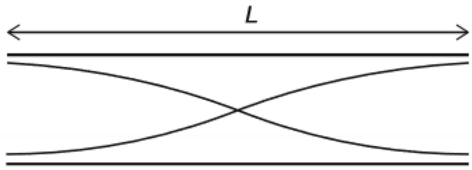


10 A pipe of length L open at both ends contains a stationary sound wave with 1 node, as shown in the diagram.



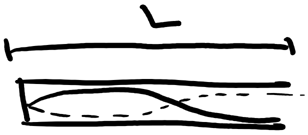
$\lambda = 2L$

$v = 8Lf$

The frequency of the stationary wave in this pipe is $4f$.

A second pipe is open at one end and closed at the other end. A stationary sound wave in this pipe contains one more node than the stationary wave shown in the diagram.

The frequency of the stationary wave in the second pipe is f.



The speed of sound is the same in both pipes.

What is the length of the second pipe?

$\frac{3\lambda}{4} = L \therefore \lambda = \frac{4L}{3}$

A $4L$

B $6L$

C $8L$

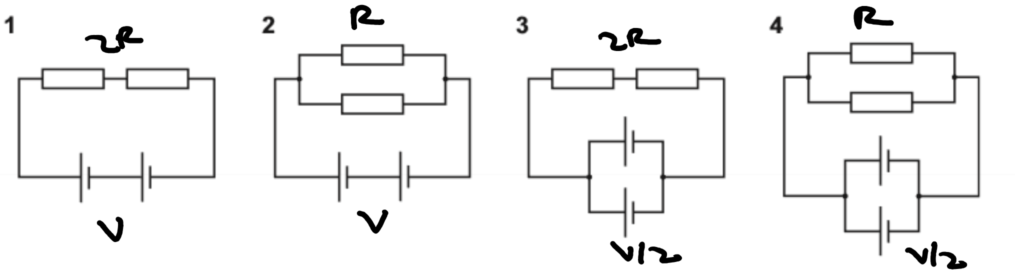
D $10L$

E $12L$

$8Lf = \frac{4L_{new}}{3} \times f$

$\therefore L_{new} = 6L$

11 The resistors in the following four circuits are identical.



The cells are identical and have no internal resistance. Each cell can supply the same total amount of energy at a constant voltage before becoming exhausted.

t_1, t_2, t_3 and t_4 are the lengths of time after which the cells in circuits **1**, **2**, **3** and **4**, respectively, become exhausted.

Which comparison of t_1, t_2, t_3 and t_4 is correct?

① $I = \frac{V}{2R}$ ③ $I = \frac{V}{4R}$

② $I = \frac{V}{R}$ ④ $I = \frac{V}{2R}$

$\therefore t_2 < t_1 = t_4 < t_3$

A $t_1 = t_2 < t_3 = t_4$

B $t_1 = t_3 < t_2 = t_4$

C $t_2 < t_1 = t_4 < t_3$

D $t_2 = t_4 < t_1 = t_3$

E $t_3 < t_1 = t_4 < t_2$

F $t_3 = t_4 < t_1 = t_2$

12 A particle of mass m is accelerated from rest by a resultant force of varying magnitude that acts in a constant direction. The kinetic energy E of the particle increases with time t according to the equation

$$E = kt$$

where k is a constant.

Which expression gives the resultant force on the particle at time T?

A k

B $2mk$

C $\sqrt{2mkT}$

D $\sqrt{\dfrac{mk}{2T}}$

E $\sqrt{\dfrac{mk}{8T}}$

F $\sqrt{\dfrac{2mk}{T}}$

G $\sqrt{\dfrac{k}{2mT}}$

$F_R = ma = m\dfrac{dv}{dt}$

$E = \frac{1}{2}mv^2 = kt$

$v = \sqrt{\dfrac{2kt}{m}}$

$\dfrac{dv}{dt} = \dfrac{1}{2}\sqrt{\dfrac{2k}{mt}} = \sqrt{\dfrac{k}{2mt}}$

$F = m\sqrt{\dfrac{k}{2mT}} = \sqrt{\dfrac{mk}{2T}}$ at time T

13 A light horizontal wire of cross-sectional area A is fixed at two points a distance $2L$ apart. The initial tension in the wire is zero.

An object of weight W is fixed directly to the centre of the wire. The wire stretches so that the object rests in equilibrium at a vertical distance of $\frac{3L}{4}$ below the original position of the wire.

What is the Young modulus of the wire?

(Assume that the wire does not exceed its limit of proportionality.)

A $\frac{2W}{A}$

B $\frac{4W}{A}$

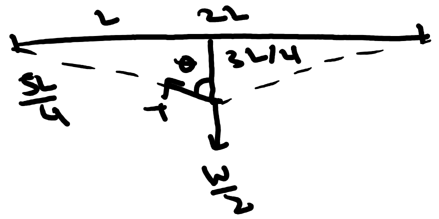
C $\frac{5W}{2A}$

D $\frac{2W}{3A}$

E $\frac{10W}{3A}$

F $\frac{20W}{3A}$

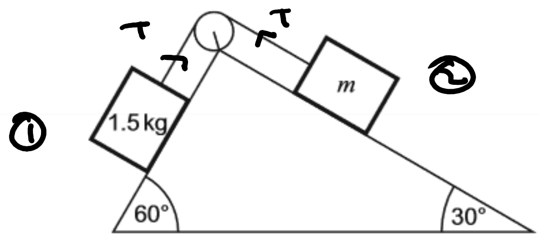
G $\frac{5W}{6A}$



$T\cos\theta = \frac{W}{2} \quad \therefore \quad T = \frac{W}{2} \div \frac{3}{5} = \frac{5W}{6}$

$E = \frac{FL}{Ax} = \frac{\frac{5W}{6} \cdot 2L}{A \cdot \frac{L}{2}} = \frac{10W}{3A}$

14 A triangular ramp with angles to the horizontal of 60° and 30° is placed with its largest face horizontal. A block of mass 1.5 kg and a block of mass m are joined by a light, inextensible string and placed on the ramp as shown.



The string passes over a light, frictionless pulley.

The maximum force of friction between the block of mass 1.5 kg and the surface of the ramp is 3.5 N.

The maximum force of friction between the block of mass m and the surface of the ramp is 5.0 N.

What is the maximum value of m that allows the blocks to remain stationary on the surfaces?

→ 2 goes →

(gravitational field strength = 10 N kg^{-1})

A 1.5 kg

B 1.65 kg

C 2.35 kg

D $\left(\frac{16\sqrt{3}}{15}\right)$ kg

E $(0.60\sqrt{3})$ kg

F $(0.30 + 1.5\sqrt{3})$ kg

G $(1.7 + 1.5\sqrt{3})$ kg

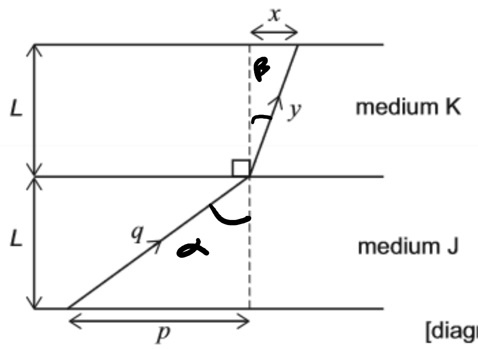
① $T = 15\sin 60 + 3.5 = \frac{15\sqrt{3}}{2} + 3.5$

② $10m\sin 30 = T + 5$

$5m = \frac{15\sqrt{3}}{2} + 3.5 + 5$

$\therefore m = \frac{15\sqrt{3}}{10} + 1.7$

15 A sound wave travels through medium J, reaches a boundary, and then travels through medium K as shown. The thickness of each medium is L.



[diagram not to scale]

The wave travels a distance q in medium J and a distance y in medium K.

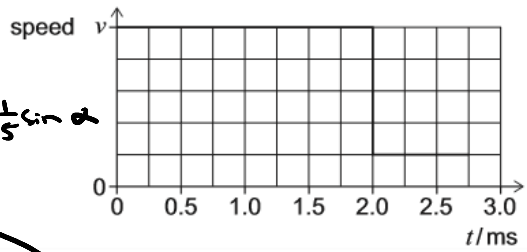
The horizontal distance travelled in medium J is p. The horizontal distance travelled in medium K is x.

The wave travels at speed v in medium J. The graph shows how the speed of the wave varies with time t as it travels distances q and y, and that the wave leaves medium K at $t = 2.75$ ms.

$n_J \sin\alpha = n_K \sin\beta$

$\frac{1}{5} = \frac{\sin\beta}{\sin\alpha} \quad \therefore \sin\beta = \frac{1}{5}\sin\alpha$

$\frac{p}{dQ} = 5 \cdot \frac{x}{dy}$



$V_J = 5V_K$

$\frac{n_J}{n_K} = \frac{c/v_J}{c/v_K} = \frac{v_K}{v_J} = \frac{1}{5}$

What is the value of $\frac{x}{p}$?

$\frac{x}{p} = \frac{1}{5} \cdot \frac{dy}{dQ}$

$\frac{dy}{dQ} = \frac{v_K \cdot 0.75}{v_J \cdot 2} = \frac{1}{5} \times \frac{3}{8} = \frac{3}{40}$

$\frac{x}{p} = \frac{3}{40} \cdot \frac{1}{5} = \frac{3}{200}$

A $\frac{3}{200}$

B $\frac{3}{40}$

C $\frac{8}{75}$

D $\frac{3}{8}$

E $\frac{8}{15}$

F $\frac{8}{3}$

G $\frac{200}{3}$

16 The drag force F acting on a sphere of radius r falling at constant speed v though air is given by

$$F = krv$$

where k is a constant.

For a sphere of uniform density and mass m falling at a constant speed, the drag force heats the surrounding air at a constant rate P.

Another sphere of the same material but with mass $8m$ falls through the air at a different constant speed.

What is the rate at which the drag force on the heavier sphere heats the surrounding air?

A $2P$

B $4P$

C $8P$

D $16P$

E $32P$

F $64P$

Rate of heating = Power = $F \times v$

$= krv^2$

ρ is the same ∴ 8m ball has $8\times$ the volume

$= 2\times$ the radius

$(V = \frac{4}{3}\pi r^3)$

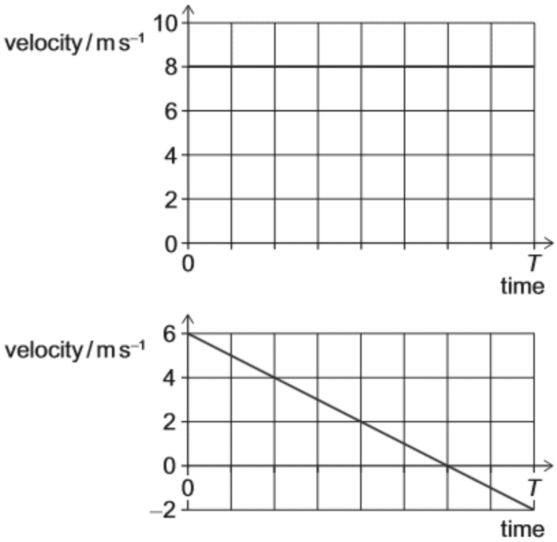
Terminal v → $mg = krv$ ∴ $v = \frac{mg}{kr}$

$\frac{8\times m}{2\times r} = 4\times v$

$P_{ratio} = 2\times r \times (4\times v)^2 = \underline{\underline{\times 32}}$

17 A projectile is launched from an inclined plane.

The graphs show the variation of the horizontal and vertical components of the velocity of the projectile with time from when it is launched until it hits the plane at time T.



What is the angle of the plane to the horizontal?

(gravitational field strength = 10 N kg^{-1})

A $\tan^{-1}\frac{1}{32}$

B $\tan^{-1}\frac{1}{8}$

C $\tan^{-1}\frac{1}{4}$

D $\tan^{-1}\frac{5}{16}$

E $\tan^{-1}\frac{1}{3}$

F $\tan^{-1}\frac{4}{3}$

Vertically:

$v^2 = u^2 + 2as$

$(-2)^2 = 6^2 - 20s$

$s = \frac{-32}{-20} = 1.6$ m above start

$(1.6 = 6t - 5t^2)5$

$25t^2 - 30t + 8 = 0$

$25t^2 - 20t - 10t + 8 = 0$

$5t(5t - 4) - 2(5t - 4) = 0$

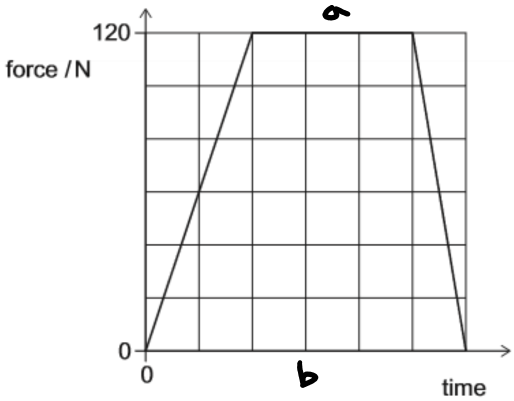
$t = 0.8$ (larger value)

Horizontal distance = 6.4 m

$\theta = \tan^{-1}\left(\frac{1.6}{6.4}\right) = \tan^{-1}\left(\frac{1}{4}\right)$ //

18 A tennis ball of mass 0.060 kg travels horizontally and strikes a vertical wall at $30\,\mathrm{m\,s^{-1}}$. It leaves the wall in the opposite direction at $20\,\mathrm{m\,s^{-1}}$.

The graph shows how the resultant horizontal force acting on the ball varies with time during this collision.



$\Delta p = 0.06(30 - -20)$
$= 0.06(50)$
$= 3\ \mathrm{Ns}$
$\frac{1}{2}(a+b)(120) = 3$
$b = 2a$
$\frac{1}{2}(3a)(120) = 3$
$a = \frac{1}{60}$
$b = \frac{1}{30}$

What is the duration of the collision?

A $\frac{1}{200}\,\mathrm{s}$

B $\frac{1}{150}\,\mathrm{s}$

C $\frac{1}{100}\,\mathrm{s}$

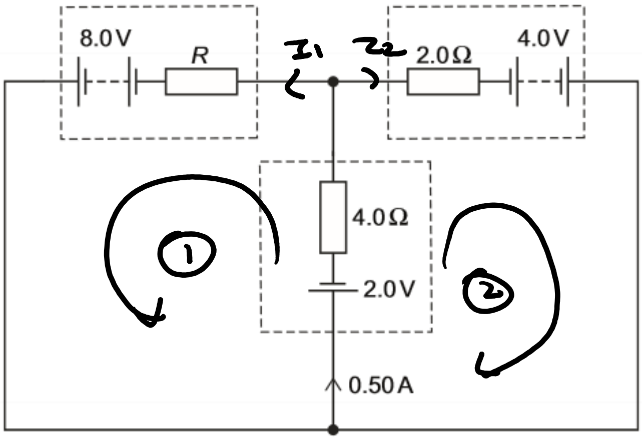
D $\frac{1}{40}\,\mathrm{s}$

E $\frac{1}{30}\,\mathrm{s}$

F $\frac{1}{20}\,\mathrm{s}$

19 A battery with an emf of 8.0 V and internal resistance R and another battery with an emf of 4.0 V and internal resistance 2.0 Ω are connected to a cell with an emf of 2.0 V and internal resistance 4.0 Ω in the circuit shown.

The current in the 2.0 V cell is 0.50 A in the direction shown in the diagram.



What is the resistance R?

A 1.6 Ω

B 2.7 Ω

C 3.2 Ω

D 8.0 Ω

E 16 Ω

① K_2: $6 = 0.5(4) + R(I_1)$

$RI_1 = 6 - 2 = \underline{4}$

② K_2: $2 = 0.5(4) + 2I_2$

$2I_2 = 0 \therefore I_2 = 0$

$\therefore I_1 = 0.5\ A$

$\therefore R = \frac{4}{0.5} = 8\ \Omega$

20 A model for how the resistivity ρ of damp soil varies with depth x from the surface is given by

$$\rho = \rho_0\left(1 - \frac{kx^2}{h^2}\right)$$

where h is the maximum depth, and k and ρ_0 are other constants.

What is the resistance of a vertical column of damp soil of cross-sectional area A and depth h?

A $\frac{\rho_0 h}{A}$

B $\frac{\rho_0 h}{A}(1-k)$

C $\frac{\rho_0 h}{2A}(2-k)$

D $\frac{\rho_0 h}{A}(1-3k)$

E $\frac{\rho_0 h}{A}\left(1-\frac{k}{3}\right)$

F $\frac{\rho_0}{A}\left(1-\frac{kh}{3}\right)$

G $\frac{\rho_0 h^2}{A}\left(\frac{1}{2}-\frac{k}{4}\right)$

$$R = \frac{\rho L}{A} = \int_0^h \underbrace{\frac{\rho_0\left(1-\frac{kx^2}{h^2}\right)}{A}\,dx}_{R \text{ for infinitesimal length } dx!}$$

$$= \frac{\rho_0}{A}\int_0^h 1 - \frac{kx^2}{h^2}\,dx$$

$$= \frac{\rho_0}{A}\left[x - \frac{kx^3}{3h^2}\right]_0^h$$

$$= \frac{\rho_0}{A}\left(h - \frac{kh}{3}\right) = \frac{\rho_0 h}{A}\left(1-\frac{k}{3}\right) //$$