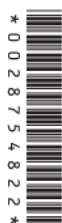




ENGINEERING ADMISSIONS ASSESSMENT D564/11

Thursday 2 November 2017

80 minutes



SECTION 1

INSTRUCTIONS TO CANDIDATES

Please read these instructions carefully, but do not open this question paper until you are told that you may do so. This paper is Section 1 of 2.

A separate answer sheet is provided for this paper. Please check you have one. You also require a soft pencil and an eraser.

Please complete the answer sheet with your candidate number, centre number, date of birth, and name.

At the end of 80 minutes, your supervisor will collect this question paper and answer sheet before giving out Section 2.

This paper contains **two** parts, **A** and **B**, and you should attempt **both** parts.

Part A	Mathematics and Physics (28 questions)
Part B	Advanced Mathematics and Advanced Physics (26 questions)

This paper contains 54 multiple choice questions. There are no penalties for incorrect responses, only marks for correct answers, so you should attempt all 54 questions. Each question is worth one mark.

Questions ask you to show your choice between options. Choose the **one** option you consider correct and record your choice on the separate answer sheet. If you make a mistake, erase thoroughly and try again.

You **must** complete the answer sheet within the time limit.

You can use the question paper for rough working, but **no extra paper** is allowed. Only your responses on the answer sheet will be marked.

Dictionaries and calculators may NOT be used.

Please wait to be told you may begin before turning this page.

This question paper consists of 42 printed pages and 2 blank pages.

PV2

© UCLES 2017

BLANK PAGE

PART A Mathematics and Physics

1 Evaluate

$$\frac{(\sqrt{12} + \sqrt{3})^2}{(\sqrt{12} - \sqrt{3})^2}$$

A 1

B 3

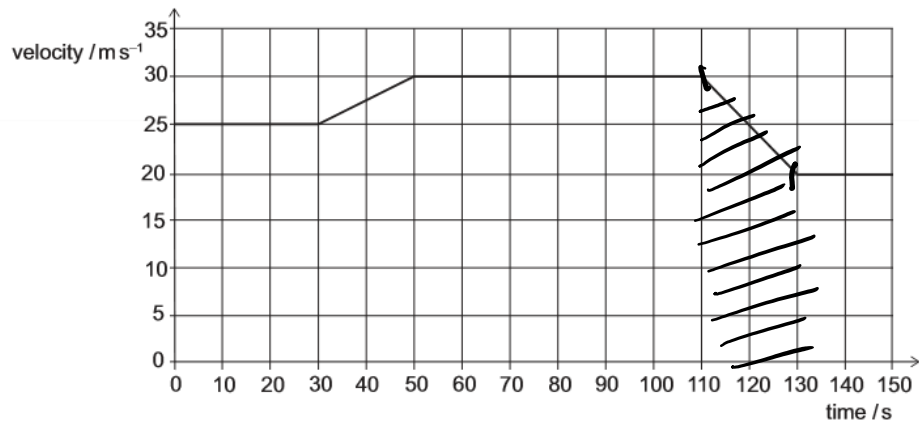
C $\frac{5}{3}$ D $\frac{7}{3}$ E $3\sqrt{3}$ ☒ F 9

$$= \frac{12 + 3 + 2\sqrt{12}\sqrt{3}}{12 + 3 - 2\sqrt{12}\sqrt{3}}$$

$$= \frac{15 + 2(6)}{15 - 2(6)} = \frac{27}{3} = 9$$

- 2 A car is travelling along a horizontal road in a straight line.

The graph is a velocity–time graph for part of the car's journey.



During this part of the journey, what is the total distance that the car travels while it is decelerating?

- A 400 m
- ☒ B 500 m
- C 550 m
- D 600 m
- E 750 m
- F 1400 m
- G 1800 m
- H 1900 m

$$\begin{aligned} \text{Distance} &= \frac{1}{2} (30 + 20) \times 20 \\ &= 10 (50) = \underline{\underline{500 \text{ m}}} \end{aligned}$$

- 3 Solve fully the inequality

$$2x^2 \geq 15 - x$$

- A $x \leq -3$
 B $x \geq 2.5$
 C $x \leq -1.5, x \geq 5$
 D $-1.5 \leq x \leq 5$
☒ E $x \leq -3, x \geq 2.5$
 F $-3 \leq x \leq 2.5$

$$\begin{aligned}
 &2x^2 + x - 15 \leq 0 \\
 &\text{Solve } 2x^2 + x - 15 = 0 \\
 &2x^2 + 6x - 5x - 15 = 0 \\
 &(2x - 5)(x + 3) = 0 \\
 &x = 5/2 \text{ or } x = -3 \\
 &\text{Graph: } \text{A parabola opening upwards with roots at } x = -3 \text{ and } x = 2.5. \text{ The region outside the roots is shaded, indicating } x \leq -3 \text{ or } x \geq 2.5.
 \end{aligned}$$

- 4 When a saucepan of water is heated from below, convection currents form and transfer heat through the liquid.

Here are three statements about the water as it is heated:

- 1 The mass of a fixed volume of the water increases. \times
 2 The density of a fixed mass of the water decreases. \checkmark
 3 The volume of a fixed mass of the water increases. \checkmark

Which of these statements help(s) to explain how convection currents are formed?

- A none of them
 B 1 only
 C 2 only
 D 3 only
 E 1 and 2 only
 F 1 and 3 only
☒ G 2 and 3 only
 H 1, 2 and 3

- 5 The equation gives y in terms of x :

$$y = 3\left(\frac{x}{2} - 1\right)^2 - 5$$

Which one of the following is a rearrangement for x in terms of y ?

A $x = 2 \pm 2\sqrt{\frac{y-5}{3}}$

B $x = 2 \pm 2\sqrt{\frac{y+5}{3}}$

C $x = 2 \pm 3\sqrt{\frac{y+5}{3}}$

D $x = -2 \pm 2\sqrt{\frac{y+5}{3}}$

E $x = -2 \pm 3\sqrt{\frac{y+5}{2}}$

F $x = 2 + 2\left(\frac{y+5}{3}\right)^2$

G $x = -2 + 2\left(\frac{y+5}{3}\right)^2$

$$\begin{aligned} \pm \sqrt{\frac{y+5}{3}} &= \frac{x}{2} - 1 \\ \frac{x}{2} &= 1 \pm \sqrt{\frac{y+5}{3}} \\ x &= 2 \pm 2\sqrt{\frac{y+5}{3}} \end{aligned}$$

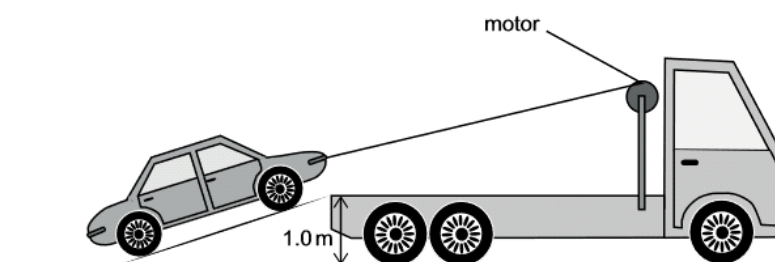
- 6 An electric motor is used to pull a broken-down car slowly from the road up a ramp on to the back of a breakdown truck.

The car has a mass of 1200 kg and is lifted through a vertical height of 1.0 m.

The total input energy to the motor is 28 kJ and it is 75% efficient.

In the process of lifting the car, energy is lost to the surroundings from the motor and from other causes.

What is the **total** energy lost to the surroundings?



(gravitational field strength = 10 N kg^{-1})

A 7.0 kJ

B 9.0 kJ

C 12 kJ

☒ D 16 kJ

E 21 kJ

F 33 kJ

$$W_d = mgh = 1200 (10) (1) = 12000 \text{ J} \rightarrow 2 \text{ efficiency}$$

$$\text{Total energy lost} = \underline{0.25(28)} + \underline{0.75(28) - 12}$$

other terms

$$= 7 + 21 - 12 = \underline{\underline{16 \text{ kJ}}}$$

- 7 A fruit stall sells apples costing £ x each, and pears costing £ y each.

Sam bought 2 apples and 5 pears, and the total cost of these was £ P .

Lesley bought 3 apples and 2 pears, and the total cost of these was £ Q .

Which of the following is an expression for the cost, in pounds (£), of a pear?

A $\frac{2Q-3P}{3}$

B $\frac{2Q-3P}{11}$

C $\frac{Q-P}{3}$

D $\frac{Q-P}{11}$

E $\frac{P-Q}{3}$

F $\frac{3P-2Q}{3}$

☒ G $\frac{3P-2Q}{11}$

$$\begin{aligned} 2x + 5y &= P & \times 3 &\rightarrow 6x + 15y = 3P \\ 3x + 2y &= Q & \times 2 &\rightarrow 6x + 4y = 2Q \\ \hline & & & 11y = 3P - 2Q \\ & & & y = \frac{3P - 2Q}{11} \end{aligned}$$

- 8 In one type of medical scanner a source is placed inside a patient's body. This source causes pairs of gamma-rays to be emitted simultaneously in opposite directions.

Detectors on each side of the patient are used to detect the gamma-rays. The distance between the two detectors is 3.0 m. When the source is at Q, half-way between the detectors, the two gamma-rays arrive at the same time.

In a particular scan the gamma-rays arrive at the two detectors with a time difference of 4.0×10^{-10} s.

Assume that, inside the patient, the gamma-rays travel at a speed of 3.0×10^8 m s⁻¹.

How far from Q, half-way between the detectors, is the gamma-ray source?

A 6.0 mm

B 12 mm

C 24 mm

☒ D 6.0 cm

E 12 cm

F 24 cm

$$\begin{aligned} \text{Difference in distance travelled} \\ \text{by both waves} &= 3 \times 10^8 \times 4 \times 10^{-10} \\ &= 12 \times 10^{-2} \text{ m} \\ &= \underline{12 \text{ cm}} \end{aligned}$$

∴ Wave 1 travels 6 cm less, and Wave 2 travels 6 cm more



- 9 P is directly proportional to Q squared.

When P is 2, Q is 4.

Q is inversely proportional to R .

When Q is 2, R is 5.

What is P in terms of R ?

A $P = \frac{5}{R}$

B $P = \frac{5}{4R}$

C $P = \frac{1}{800R^2}$

D $P = \frac{5}{4R^2}$

☒ E $P = \frac{25}{2R^2}$

F $P = \frac{800}{R^2}$

G $P = \frac{R^2}{50}$

H $P = \frac{25R^2}{2}$

$$P = kQ^2$$

$$2 = 16k \Rightarrow k = \frac{1}{8}$$

$$P = \frac{1}{8}Q^2$$

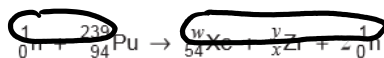
$$Q = \frac{10}{R}$$

$$2 = \frac{k}{5} \Rightarrow k = 10$$

$$Q = \frac{10}{R}$$

$$P = \frac{1}{8} \left(\frac{10}{R} \right)^2 = \frac{100}{8R^2} = \frac{25}{2R^2}$$

- 10 When a plutonium-239 nucleus absorbs a neutron it undergoes nuclear fission. One particular fission reaction results in the creation of xenon and zirconium as daughter nuclei. The nuclear equation for this reaction is shown but with some non-zero integers replaced by the letters w , x , y and z .



Which equation is correct?

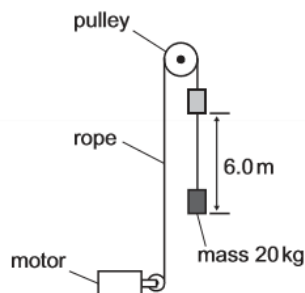
- A $w + y = 240$ ~~x~~ $w + y = 239$
 B $z = 240 - (w + y)$ $\rightarrow 240 = w + y + z$
 C $x = 40 - z$ ~~x~~
 D $94 = 54 + x + 1$ ~~x~~
 E $240 = 54 + x$ ~~x~~
 F $94 = w + y + 1$ ~~x~~

- 11 Which one of the following is a simplification of

$$2 - \frac{x^2(9x^2 - 4)}{x^3(2 - 3x)}$$

- A $-1 - \frac{2}{x}$
 B $-1 + \frac{2}{x}$
 C $5 - \frac{2}{x}$
 D $5 + \frac{2}{x}$
 E $5 - \frac{3}{x}$
 F $5 + \frac{3}{x}$
- $$= \frac{2 + \cancel{x^2}(3x+2)(3x+2)}{\cancel{x^3}(2-3x)}$$
- $$= 2 + \frac{3x+2}{x}$$
- $$= 5 + \frac{2}{x}$$

- 12 An electric motor is connected to a constant 12 V d.c. supply. The motor is used to lift a mass of 20 kg by means of a rope and pulley. The mass is lifted vertically through a height of 6.0 m in a time of 5.0 s. The complete lifting system (motor, rope and pulley) is 80% efficient.



What is the current in the electric motor?

(gravitational field strength = 10 N kg^{-1})

A 1.6 A

B 2.0 A

C 2.5 A

D 16 A

E 20 A

F 25 A

$$\text{Power out} = \frac{20 \times 10 \times 6}{5} = 240 \text{ W}$$

$$\text{Power in} = \frac{240}{0.8} = 300 \text{ W}$$

$$VI = 300 \therefore I = \underline{\underline{25 \text{ A}}}$$

- 13 What is the value of x that makes the following expression correct?

$$2^{3+2x} 4^x 8^{-x} = 4\sqrt{2}$$

A -2.25

B -1.75

C -1.5

D -0.5

E -0.25

$$2^{3+2x} 2^{2x} 2^{-3x} = 2^{2.5}$$

$$2^{3+x} = 2^{2.5}$$

$$x = 2.5 - 3 = -0.5$$

- 14 The nuclide P_QX decays to the stable nuclide Y. During this process four particles are emitted: an α -particle and three β^- particles.

Which of the following is **not** a nuclide that could be formed at any stage during this process?

nuclide	atomic mass	atomic number
A	P	Q-1
B	P	Q+1
C	P	Q+2
D	P	Q+3
E	P-4	Q-2
F	P-4	Q-1
G	P-4	Q
H	P-4	Q+1

✓ β
 ✓ β, β
 ✓ β, β, β
 ✓ α
 ✓ β, α
 ✓ β, β, α
 ✓ $\beta, \beta, \beta, \alpha$

- 15 There are 100 students in Year 10.

Each student studies exactly one of French, German, and Spanish.

X girls study French and there are 3X girls in total.

2Y boys study German.

There are 35 students studying Spanish of which Y are boys.

Which of the following is an expression for the total number of students studying German?

- A $X + 2Y$
 B $X + Y + 35$
 C $X + 3Y - 35$
 D $2X + 2Y$
 E $2X + Y - 35$
F $2X + 3Y - 35$
 G $2X + Y + 35$

	F	G	S	T
B		2Y	Y	100-3X
G	X		35-Y	3X
T			35	100

$$\text{Girls, German} = 3X - X - (35 - Y)$$

$$= 2X + Y - 35$$

$$2X + Y - 35 + 2Y = 2X + 3Y - 35$$

- 16 The radius of an iron-56 atom is 3.0×10^4 times greater than the radius of an iron-56 nucleus.

What is the value of $\frac{\text{density of an iron atom}}{\text{density of an iron nucleus}}$? *Same mass*

- ☒ A $(3.0 \times 10^4)^{-3}$
 B $(3.0 \times 10^4)^{-2}$
 C $(3.0 \times 10^4)^{-1}$
 D $(3.0 \times 10^4)^1$
 E $(3.0 \times 10^4)^2$
 F $(3.0 \times 10^4)^3$

$$\frac{V_{\text{atom}}}{V_{\text{nucleus}}} = (3 \times 10^4)^3 \rightarrow \text{spherical}$$

$$\therefore \frac{\rho_{\text{atom}}}{\rho_{\text{nucleus}}} = (3 \times 10^4)^{-3}$$

- 17 An exterior angle of a regular polygon with n sides is 4° larger than an exterior angle of a regular polygon with $(n+3)$ sides.

What is the value of n ?

- A 10
 B 12
☒ C 15
 D 18
 E 21
 F 24
 G 27

$$\text{ext. angle of polygon} = \frac{360}{n}$$

$$\frac{360}{n} - \frac{360}{n+3} = 4$$

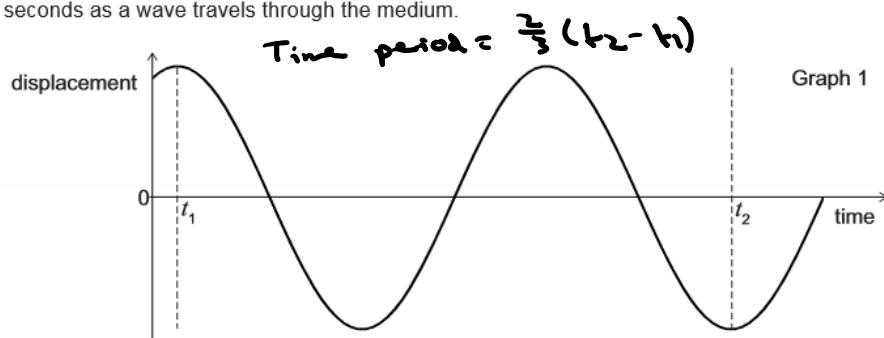
$$360n + 1080 - 360n = 4n(n+3)$$

$$n(n+3) - 270 = 0$$

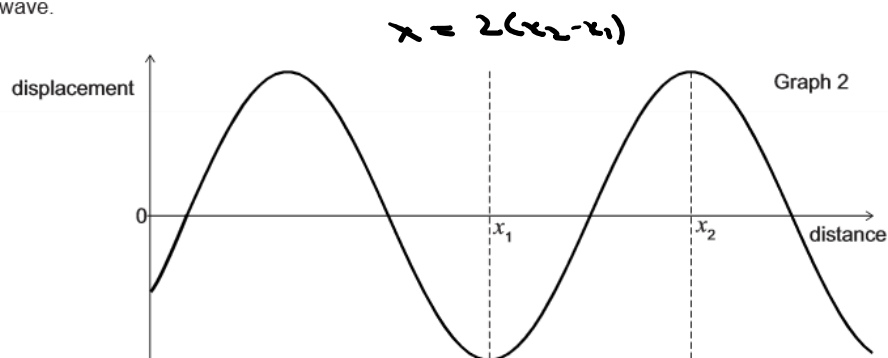
$$n^2 + 3n - 270 = 0$$

$$(n+18)(n-15) = 0 \therefore n = 15$$

- 18 Graph 1 shows how the displacement of one of the particles of a medium varies with time in seconds as a wave travels through the medium.



Graph 2 shows how the displacement varies with distance in metres at one time for the same wave.



Which expression gives the speed in m s^{-1} of the wave?

A $\frac{4(x_2 - x_1)}{3(t_2 - t_1)}$

B $\frac{3(x_2 - x_1)}{2(t_2 - t_1)}$

C $\frac{2(x_2 - x_1)}{t_2 - t_1}$

D $\frac{8(x_2 - x_1)}{3(t_2 - t_1)}$

E $\frac{3(x_2 - x_1)}{t_2 - t_1}$

F $\frac{6(x_2 - x_1)}{t_2 - t_1}$

$$v = f\lambda = \frac{\lambda}{T} = \frac{2(x_2 - x_1)}{\frac{2}{3}(t_2 - t_1)} = \frac{3(x_2 - x_1)}{t_2 - t_1}$$

- 19 The bearing of a ship R from a lighthouse L is 220°

A canoe C is due North of R .

C is the same distance from the ship and the lighthouse.

What is the bearing of L from C ?

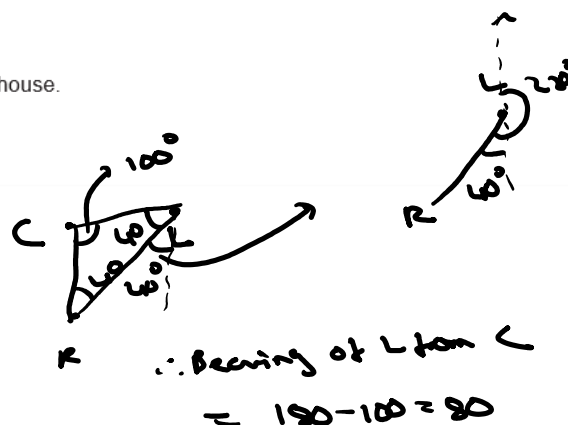
A 070°

☒ B 080°

C 090°

D 100°

E 140°



- 20 A kettle is designed to work from a car's power socket. The kettle has a power rating of 150 W when a constant voltage of 12.0 V d.c. is applied across its element.

How much charge passes through the element of this kettle when the voltage of 12.0 V is applied across it for 20 minutes?

A 96 C

B 250 C

☒ C 15000 C

D 36000 C

E 900000 C

F 2160000 C

$$I = \frac{150}{12} = \frac{50}{4} = 12.5\text{ A}$$

$$Q = 12.5 \times 20 \times 60$$

$$= 1250 \times 12 = 12500 + 2500$$

- 21 The hands of a 12-hour analogue clock move continuously. When the time on the clock is $4:00$, the angle between the minute hand and the hour hand is 120° .

What is the angle between the two hands at $4:40$?

A 80°

☒ B 100°

C 110°

D 120°

E 140°

Angle between 8 and 4 = 120°

Hour hand has moved $\frac{2}{3} \times \frac{360}{12}^\circ$

$$= 20^\circ$$

$$120 - 20 = 100$$

- 22 A freight train travelling on a straight horizontal track at 2.0 m s^{-1} collides with a passenger train travelling at 5.0 m s^{-1} in the opposite direction. Both trains immediately come to a complete stop on the track.

The freight train has three locomotives of 130 tonnes each and seven container wagons of 30 tonnes each. The passenger train has two locomotives of 70 tonnes each and a number of passenger carriages of 10 tonnes each.

How many passenger carriages does the passenger train have?

A 7

B 9

☒ C 10

D 24

E 46

$$\text{COM: } [3(130) + 7(30)]2 = [140 + 10n]5$$

$$1200 = 5(140 + 10n)$$

$$140 + 10n = 240$$

$$10n = 100$$

- 23 A pet shop has 4 female rabbits and x male rabbits for sale.

A customer buys 2 of the rabbits, chosen at random, and each rabbit is equally likely to be chosen.

The probability that both the chosen rabbits are male is $\frac{1}{3}$.

What is the value of x ?

A 2

B 4

☒ C 6

D 8

E 9

F 11

G 12

$$\frac{x}{x+4} \times \frac{x-1}{x+3} = \frac{1}{3}$$

$$3(x^2 - x) = x^2 + 7x + 12$$

$$2x^2 - 10x - 12 = 0$$

$$x^2 - 5x - 6 = 0$$

$$(x-6)(x+1) = 0$$

$$x > 0 \therefore x = \underline{6}$$

- 24 Consider the following three statements about a parachutist of mass 72 kg falling vertically at a constant velocity of 5.0 m s^{-1} after the parachute has opened:

- 1 The parachutist has a constant kinetic energy of 1800 J.
- 2 The parachutist is losing gravitational potential energy at a rate of 3600 J s^{-1} .
- 3 Air resistance and the force of gravity acting on the parachutist are a Newton's third law pair of forces.

Which of the statements is/are correct?

(gravitational field strength = 10 N kg^{-1})

A none of them

B 1 only

☒ C 2 only

D 3 only

E 1 and 2 only

F 1 and 3 only

G 2 and 3 only

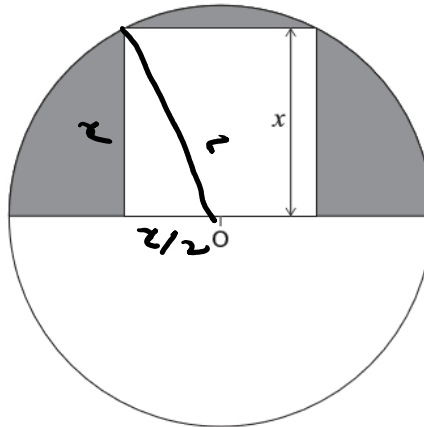
H 1, 2 and 3

$$KE = 36 \times 25 \neq 1800$$

$$72 \times 5 \times 10 = 3600 \text{ J s}^{-1} \text{ loss}$$

$$\text{of GPE/s}$$

25



The diagram shows a square with side of length x cm. A circle is drawn with centre O which lies at the mid-point of one of the sides of the square. This side forms part of a diameter of the circle. The circle passes through two corners of the square as shown.

What is the area, in cm^2 , of the shaded part of the semi-circle?

A $(\pi - 1)x^2$

B $\left(\frac{\pi - 2}{2}\right)x^2$

C $\left(\frac{3\pi - 2}{2}\right)x^2$

D $\left(\frac{3\pi - 4}{4}\right)x^2$

E $\left(\frac{5\pi - 4}{4}\right)x^2$

☒ F $\left(\frac{5\pi - 8}{8}\right)x^2$

$\text{Square area} = x^2$
 $r^2 = x^2 + \frac{x^2}{4} = \frac{5x^2}{4} \therefore r = \frac{x\sqrt{5}}{2}$
 $\text{Shaded Area} = \frac{\pi r^2}{2} - x^2 = \frac{5\pi x^2}{8} - x^2$

- 26 Two radioactive sources X and Y have half-lives of 3.0 hours and 2.0 hours respectively. The product of the decay of both of the sources is a stable isotope of the element Z.

Six hours ago a mixture contained the same number of atoms of both X and Y, and no other atoms.

What fraction of the mixture is now made up of atoms of Z?

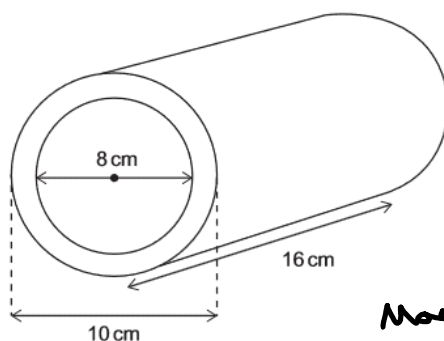
- A $\frac{10}{16}$
 B $\frac{11}{16}$
 C $\frac{12}{16}$
 D $\frac{13}{16}$
 E $\frac{14}{16}$
 F $\frac{15}{16}$

	<u>T</u>	<u>X</u>	<u>Y</u>	<u>Z</u>
0	1	1	1	0
1	1	1	$\frac{1}{2}$	$\frac{1}{2}$
2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$
3	1	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{7}{8}$
4	1	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{15}{16}$
5	1	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{31}{32}$
6	1	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{63}{64}$

$\frac{6}{4} + \frac{1}{8} = \frac{13}{8}$
 $\frac{13}{8} \div 2 = \frac{13}{16}$ (Fraction that is Z)

- 27 A cylindrical hollow metal pipe is 16 cm long.
 It has an external diameter of 10 cm and an internal diameter of 8 cm.
 The density of the metal from which the pipe is made is 8 grams per cm^3 .

[diagram not to scale]



What is the mass of the pipe in grams?

- A 8π
 B 16π
 C 18π
 D 72π
 E 128π
 F 512π
 G 1152π
 H 4608π

$$\begin{aligned}
 \text{Mass} &= [\pi(5^2) - \pi(4^2)] \cdot 16 \cdot \rho \\
 &= 9\pi \times 16 \times 8 \\
 &= 72\pi \times 16 = \underline{\underline{1152\pi}}
 \end{aligned}$$

- 28 Car X passes car Y on a motorway.

Car X is travelling at 1.5 times the speed of car Y. $= \frac{9}{4} \times \text{KE}$

The mass of car X is $\frac{4}{5}$ of the mass of car Y.

$$\frac{4}{5} \times \frac{9}{4} = \frac{9}{5} \quad (\text{as KE} \propto v^2 \text{ or } m)$$

How do the kinetic energies of the two cars compare?

- A kinetic energy of car X = 0.90 × kinetic energy of car Y
- B kinetic energy of car X = 0.96 × kinetic energy of car Y
- C kinetic energy of car X = 1.20 × kinetic energy of car Y
- D kinetic energy of car X = 1.44 × kinetic energy of car Y
- E** kinetic energy of car X = 1.80 × kinetic energy of car Y

PART B Advanced Mathematics and Advanced Physics

NOTE: questions in this part that are not covered by the ESAT content specification are indicated by a cross through the question number.

29 Which one of the following is a simplification of

$$1 - \left(\frac{3 + \sqrt{3}}{6 - 2\sqrt{3}} \right)^2$$

A $-\frac{3}{4}$

B $\frac{3}{4}$

C $-\frac{3}{4} - \frac{\sqrt{3}}{7}$

D $\frac{3}{4} - \frac{\sqrt{3}}{7}$

☒ E $-\frac{3}{4} - \sqrt{3}$

F $\frac{3}{4} - \sqrt{3}$

G $-\frac{\sqrt{3}}{2}$

H $\frac{\sqrt{3}}{2}$

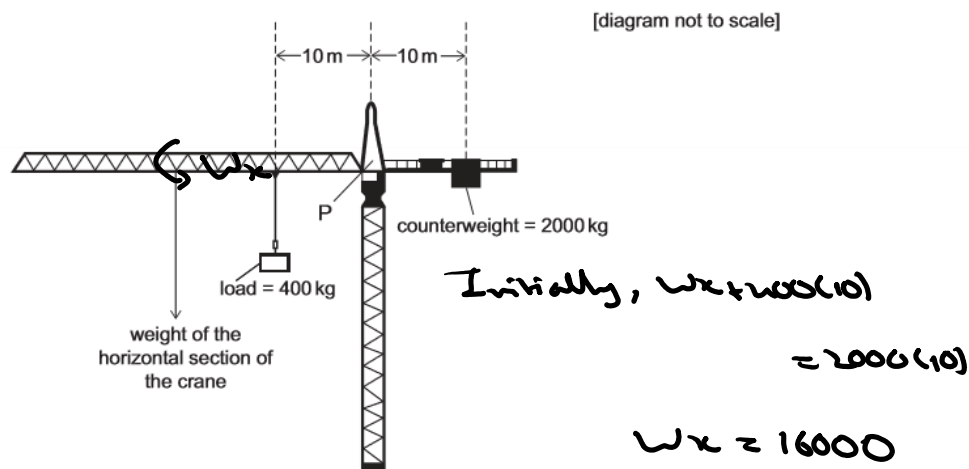
$$\frac{3+\sqrt{3}}{6-2\sqrt{3}} \times \frac{6+2\sqrt{3}}{6+2\sqrt{3}} = \frac{24+12\sqrt{3}}{24} = 1 + \frac{\sqrt{3}}{2}$$

$$a^2 - b^2 = (a+b)(a-b)$$

$$\Rightarrow \left(1 + 1 + \frac{\sqrt{3}}{2}\right) \left(1 - 1 - \frac{\sqrt{3}}{2}\right)$$

$$= \left(2 + \frac{\sqrt{3}}{2}\right) \left(-\frac{\sqrt{3}}{2}\right) = -\frac{\sqrt{3}}{4} - \sqrt{3}$$

- ✗ The diagram shows a crane being used on a building site. The crane is perfectly balanced about P.



The load is now moved to the left by 5.0 m.

To keep the crane perfectly balanced about P, how far does the counterweight have to move, and in which direction?

(gravitational field strength = 10 N kg^{-1})

- A 1.0 m to the left
- ☒ B 1.0 m to the right
- C 3.0 m to the left
- D 3.0 m to the right
- E 4.0 m to the left
- F 4.0 m to the right

Now, $Wx + 400(15) = 2000(x)$

$$x = \frac{22000}{2000} = 11 \text{ m}$$

- 31 k is the smallest positive value of x which is a solution to **both** the equations $2\sin x + 1 = 0$ and $2\cos 2x = 1$ 210°

How many values of x in the range $0 \leq x \leq k$ are solutions to at least one of these equations?

A 0

B 2

☒ C 3

D 4

E 8

$$2\sin x + 1 = 0$$

$$\sin x = -\frac{1}{2}$$

$$x = 210^\circ, 330^\circ + 360^\circ$$

$$\therefore x = 30^\circ, 150^\circ, 210^\circ$$

$$2\cos 2x = 1$$

$$\cos 2x = \frac{1}{2}$$

$$2x = 60^\circ, 300^\circ + 360^\circ$$

$$x = 30^\circ, 150^\circ + 180^\circ$$

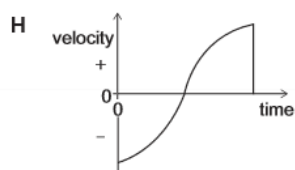
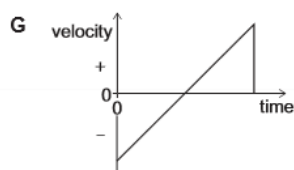
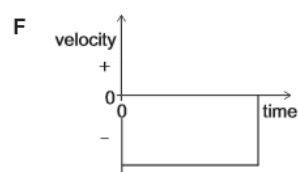
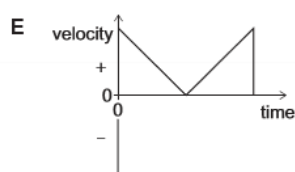
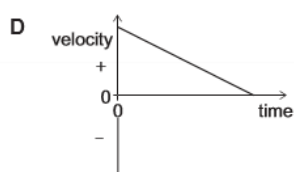
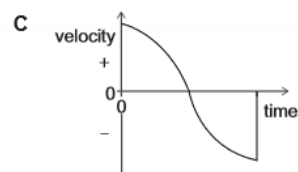
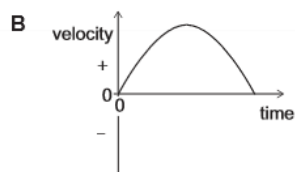
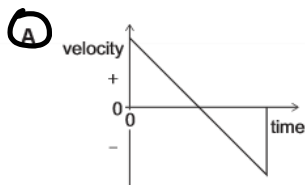
- ✗ A ball is thrown vertically upwards in still air and is then caught at the same height when it comes back down.

Constant acceleration

Which velocity–time graph shows this complete motion?

(Take upwards as positive, and ignore air resistance.)

*v ↑ initially then ↓
+ve -ve*



33 Which of the following is a solution to the equation $3^{(2x+1)} - 6(3^x) = 0$?

A $\log_2 3$

☒ B $\log_3 2$

C 2

D $\log_{10} 2$

E $\frac{2}{3}$

$$\text{Let } v = 3^x$$

$$3(3^{2x}) - 6(3^x) = 0$$

$$3v^2 - 6v = 0 \Rightarrow 3v(v-2) = 0$$

$$\cancel{v=0} \text{ or } v=2$$

$$3^x \neq 0 \quad 3^x = 2 \therefore x = \log_3 2$$

☒ An aircraft is climbing at constant speed in a straight line at an angle of 10° to the horizontal.

Which statement about the resultant force on the aircraft is correct?

A It is parallel to its motion.

B It is perpendicular to its motion.

☒ C It is zero. **N1**

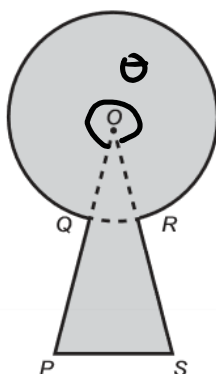
D It is equal to its weight.

E It is equal to the drag acting on the aircraft.

- 35 The diagram shows the outline of a keyhole consisting of three straight sides and an arc from a circle.

The sides PQ and RS are both 18 mm in length and when extended meet at the centre of the circle O forming an angle of $\frac{\pi}{6}$ radians.

The longer arc from Q to R has length 22π mm.



[diagram not to scale]

$$\frac{\theta}{2\pi} \times 2\pi r = 22\pi$$

$$r = 22\pi \div \frac{11\pi}{6} = 12 \text{ mm}$$

$$\text{Area} = \frac{\theta}{2\pi} \times \pi r^2 + \frac{1}{2}(\theta)(w) \quad \text{c.s.} \quad (360-\theta)$$

What is the area, in mm^2 , of the keyhole as shaded in the diagram?

A $121\pi + \frac{841}{4}$

B $121\pi + \frac{841\sqrt{3}}{4}$

☒ C $132\pi + 225$

D $132\pi + 225\sqrt{3}$

E $144\pi + 225$

F $144\pi + 225\sqrt{3}$

$$= \frac{11\pi}{12}(12^2) + \frac{1}{2}(360-\theta) \text{ c.s.}$$

$$= 132\pi + 225$$

- ✗ A horizontal, uniform bar of mass 60 kg is 4.0 m long and is pivoted at one end. The bar is held in equilibrium by a force F at the other end of the bar, acting at an angle of 60° to the horizontal.

[diagram not to scale]



Which expression gives the magnitude of F in newtons?

(gravitational field strength = 10 N kg^{-1})

- A $\frac{30}{\sin 60^\circ}$
- B $\frac{30}{\cos 60^\circ}$
- C $\frac{60}{\sin 60^\circ}$
- D $\frac{60}{\cos 60^\circ}$
- ☒ E $\frac{300}{\sin 60^\circ}$
- F $\frac{300}{\cos 60^\circ}$
- G $\frac{600}{\sin 60^\circ}$
- H $\frac{600}{\cos 60^\circ}$
- Handwritten notes:
- $$\tau_{\text{pivot}}: 600(2) = F \sin 60(4)$$
- $$F = \frac{1200}{2\sqrt{3}} = \frac{300}{\sin 60}$$

- 37 It is given that $y = 8^p$ and $z = \left(\frac{1}{2}\right)^{2q}$ where p and q are real numbers.

Which of the following expressions is a simplification of $\log_2\left(\frac{y^3}{z^2}\right)$?

A $6p - 4q$

B $6p + 4q$

C $6p - 8q$

D $6p + 8q$

E $9p - 4q$

☒ F $9p + 4q$

G $9p - 8q$

H $9p + 8q$

$$\log_2 y = \log_2 2^{3p} = 3p$$

$$\log_2 y^3 = 9p$$

$$\log_2 z = \log_2 2^{-2q} = -2q$$

$$\log_2 z^2 = -4q$$

$$\begin{aligned} \log_2\left(\frac{y^3}{z^2}\right) &= \log_2 y^3 - \log_2 z^2 \\ &= 9p - (-4q) \end{aligned}$$

- ☒ A ball starts at a speed of 40.0 m s^{-1} . The ball is subject to a constant deceleration of 14.4 m s^{-2} as it travels a distance of 20.0 m in a straight line.

What is the final speed of the ball?

A 16.0 m s^{-1}

B 20.0 m s^{-1}

C 25.6 m s^{-1}

D 32.0 m s^{-1}

☒ E 36.2 m s^{-1}

F 39.3 m s^{-1}

$$v^2 = u^2 + 2as$$

$$v^2 = 1600 + 2(-14.4)(20)$$

$$= 1600 - 288 = 1312$$

$$v = \sqrt{1312} \approx 36.2 \text{ m/s}$$

- 39 The graph of the function $y = x^3 + px^2 + qx + 6$, where p and q are real constants, has a local maximum when $x = 2$ and a local minimum when $x = 4$.

What are the values of p and q ?

- A $p = -3$ and $q = -8$
 B $p = -3$ and $q = 8$
 C $p = 3$ and $q = -8$
 D $p = -9$ and $q = 24$
 E $p = 9$ and $q = 24$
 F $p = 9$ and $q = -24$

$$y = x^3 + px^2 + qx + 6$$

$$y'(2) = y'(4) = 0$$

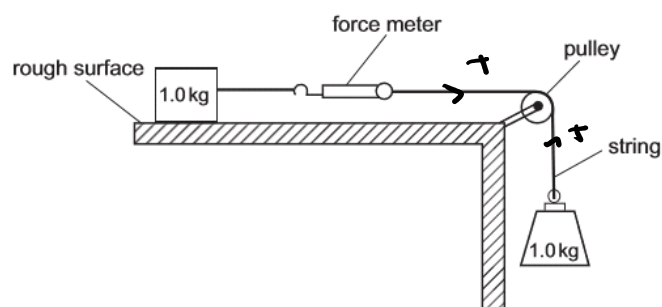
$$12 + 4p + q = 0$$

$$-48 + 8p + q = 0$$

$$-36 - 4p = 0 \therefore p = -9$$

$$q = -12 - 4p = 24$$

- ✗ A block of mass 1.0 kg is at rest on a rough horizontal surface. The block is attached by a light inextensible string to a force meter. The other end of the force meter is attached by another light inextensible string via a frictionless pulley to a load of mass 1.0 kg . The block remains stationary.



What is the reading on the force meter?

(gravitational field strength = 10 N kg^{-1})

- A 0.0 N
- B 0.5 N
- C 1.0 N
- D 2.0 N
- E 5.0 N
- F 10 N**
- G 20 N

$$T_{\text{string}} = \underline{\underline{10\text{ N}}}$$

$$\therefore F_{\text{meter also}} = \underline{\underline{10\text{ N}}}$$

41 In triangle PQR

$$PQ = 4x \text{ cm}$$

$$QR = (8 - 3x) \text{ cm}$$

$$\angle PQR = 60^\circ$$

What is the maximum value of the area, in cm^2 , of triangle PQR ?

A $\frac{8\sqrt{3}}{3}$

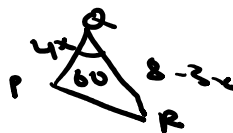
B $\frac{16}{3}$

C $\frac{69\sqrt{3}}{16}$

D $\frac{16\sqrt{3}}{3}$

E $\frac{32}{3}$

F $\frac{32\sqrt{3}}{3}$



$$\begin{aligned} \text{Area} &= 8\sqrt{3}\left(\frac{4}{3}\right) - 3\sqrt{3}\left(\frac{16}{9}\right) \\ &= \frac{32\sqrt{3}}{3} - \frac{16\sqrt{3}}{3} \end{aligned}$$

$$\text{Area} = \frac{1}{2}(4x)(8-3x)\sin 60$$

$$= 2x(8-3x)\left(\frac{\sqrt{3}}{2}\right)$$

$$= \sqrt{3}x(8-3x) = 8\sqrt{3}x - 3\sqrt{3}x^2$$

$$\frac{dA}{dx} = 8\sqrt{3} - 6\sqrt{3}x \therefore x = \frac{4}{3}$$

~~X~~ An apple of mass 100 g, growing on a tree, falls vertically from a height of 4.0 m above the ground. It hits the ground with a speed of 8.0 m s^{-1} .

How much work does the apple do against resistive forces during its descent, before it hits the ground?

(gravitational field strength = 10 N kg^{-1})

A 0.80 J

B 3.6 J

C 4.0 J

D 7.2 J

E 8.0 J

$$\text{GPE} = 0.1(10)(4) = 4$$

$$\text{KE} = 0.05(64)$$

$$= 3.2$$

$$4 - 3.2 = 0.8$$

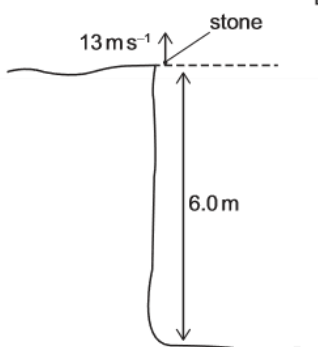
- 43 Given that $y = (2+3x)^6$, what is the coefficient of x^3 in $\frac{dy}{dx}$?

A 240
B 4320
C 4860
D 12960
☒ E 19440

$$\begin{aligned} y &= 6(3)(2+3x)^5 = 18(2+3x)^5 \\ x^3 \text{ coefficient} &= 18(2^2)(3^3)(5C_3) \\ &= 180(4)(27) \\ &= 720 \times 27 \\ &= 19440 \end{aligned}$$

- ☒ A stone is fired vertically upwards at a speed of 13 m s^{-1} on a still day from the top of a 6.0 m high cliff. It then falls down and lands at the bottom of the cliff.

[diagram not to scale]



From when the stone passes the top of the cliff on the way down, how long does it take to reach the ground at the bottom of the cliff?

(air resistance can be ignored; gravitational field strength = 10 N kg^{-1})

☒ A 0.40 s
B $\frac{6.0}{6.5} \text{ s}$
C 0.60 s
D $\sqrt{1.2} \text{ s}$
E 1.3 s
F 2.0 s
G 2.5 s
H 3.0 s

$$\begin{aligned} s &= ut + \frac{1}{2}at^2 \\ -6 &= 13t - 5t^2 \\ 5t^2 - 13t - 6 &= 0 \\ (5t+2)(t-3) &= 0 \\ 0 &= 13t - 5t^2 \\ t &= 13/5 = 2.6 \text{ s} \\ 3 - 2.6 &= 0.4 \end{aligned}$$

- 45 A geometric progression has first term equal to 1 and common ratio $\frac{1}{2}\sin 2x$

The sum to infinity of the series is $\frac{4}{3}$

Find the possible values of x in the range $\pi \leq x \leq 2\pi$

A $\frac{13}{12}\pi, \frac{17}{12}\pi$

B $\frac{7}{6}\pi, \frac{4}{3}\pi$

C $\frac{7}{6}\pi, \frac{11}{6}\pi$

D $\frac{5}{4}\pi, \frac{7}{4}\pi$

E there are no values of x in this range

$$\frac{4}{3} = \frac{1}{1-r}$$

$$4 - 4r = 3$$

$$4r = 1$$

$$r = \frac{1}{4} = \frac{1}{2}\sin 2x$$

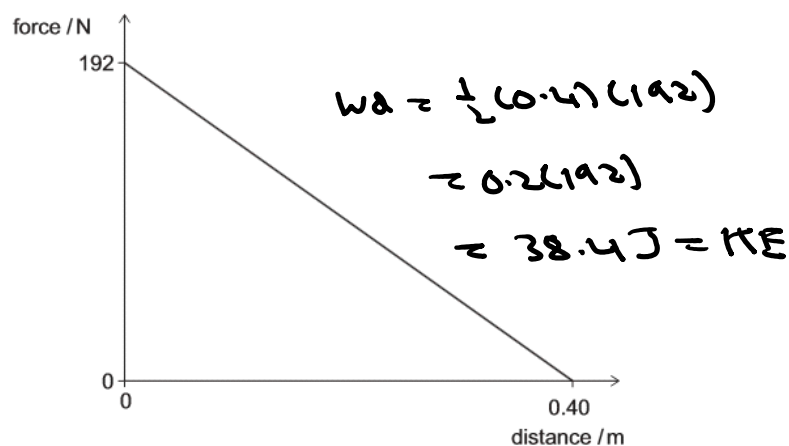
$$\sin 2x = \frac{1}{2}$$

$$2x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$$

$$\text{In range, } x = \frac{13\pi}{12}, \frac{17\pi}{12}$$

- ✗ An archer fires an arrow of mass 0.024 kg vertically upwards from a bow.

The graph shows how the force of the bowstring on the arrow varies with distance as the arrow moves upwards.



The work done by the force of the bowstring is given by the area under the force-distance graph.

When the arrow leaves the bow, what is the kinetic energy of the arrow, and what is the maximum height that it gains from this point?

(Air resistance can be ignored. The effect of gravity as the arrow is fired is negligible compared to the force of the bowstring. The gravitational field strength = 10 N kg^{-1} .)

	kinetic energy / J	height / m
A	38.4	16
B	38.4	160
C	38.4	1600
D	38.4	16 000
E	76.8	32
F	76.8	320
G	76.8	3200
H	76.8	32 000

$$mgh = 38.4$$

$$h = \frac{38.4}{10(0.024)} = \frac{38.4}{0.24} = 160 \text{ m}$$

- 47 The sequence of numbers $u_1, u_2, u_3, \dots, u_n, \dots$ is given by

$$u_1 = 2$$

$$u_{n+1} = pu_n + 3$$

where p is an integer.

The fourth term, u_4 , is equal to -7

What is the value of $u_1 + u_2 + u_3 + u_4$?

A -10

B -2

☒ C -1

D 8

E 26

$$u_2 = 2p + 3$$

$$u_3 = 2p^2 + 3p + 3$$

$$u_4 = 2p^3 + 3p^2 + 3p + 3 = -7$$

$$2p^3 + 3p^2 + 3p + 10 = 0$$

$$f(-2) = -16 + 12 - 6 + 10 = 0$$

$$\Rightarrow (p+2)(2p^2 + ap + b)$$

$$b = 5$$

$$2a + b = 3 \Rightarrow a = -1$$

$$\Rightarrow (p+2)(2p^2 - p + 5)$$

$$= \underline{(p+2)} (\text{Non-integer } p)$$

$$u_1 = 2$$

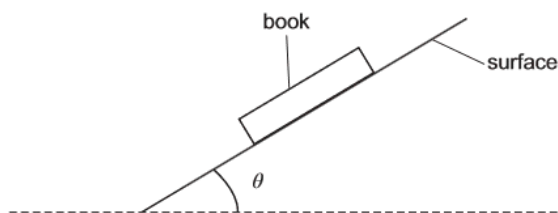
$$u_2 = -1$$

$$u_3 = 5$$

$$u_4 = -7$$

$$\text{Sum} = -1$$

- ✗ A book of mass m rests on a rough horizontal surface. The surface is now tilted as shown:



When the angle of tilt θ is 20° , the book slides down the slope at a constant speed.

What is the acceleration of the book down the slope when the angle of tilt is 25° ?

(gravitational field strength = g)

A $g (\cos 20^\circ - \sin 20^\circ \tan 5^\circ)$

B $g (\cos 20^\circ - \sin 20^\circ \tan 25^\circ)$

C $g (\cos 25^\circ - \sin 5^\circ \tan 20^\circ)$

D $g (\cos 25^\circ - \sin 25^\circ \tan 20^\circ)$

E $g (\sin 20^\circ - \cos 20^\circ \tan 5^\circ)$

F $g (\sin 20^\circ - \cos 20^\circ \tan 25^\circ)$

G $g (\sin 25^\circ - \cos 5^\circ \tan 20^\circ)$

☒ H $g (\sin 25^\circ - \cos 25^\circ \tan 20^\circ)$

Friction = $mg \sin 20$

$\mu mg \cos 20 = mg \sin 20$

$\mu = \tan 20$

Now:

$mg \sin 25 - \mu mg \cos 25 = ma$

$a = g (\sin 25 - \tan 20 \cos 25)$

- 49 Find the complete set of values of x for which

$$\frac{x^3 - 6x^2 + 9x - 4}{x} > 0$$

☒ A $x < 0, x > 4$

B $0 < x < 4$

C $0 < x < 1, x > 4$

D $x < 0, 1 < x < 4$

E $x < 1, x > 4$

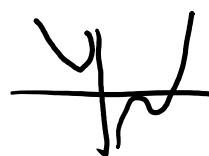
F $1 < x < 4$

$\frac{x^3 - 6x^2 + 9x - 4}{x} = 0$

$x^3 - 6x^2 + 9x - 4 = 0$

$(x-1)(x^2 - 5x + 4) = 0$

$x = 1, 1, 4$



> 0 for $x < 0$
or
 $x > 4$

- ✗ A suitcase of mass m is on a conveyor belt which moves upwards at a constant speed at an angle of θ to the horizontal. The coefficient of friction between the suitcase and the slope is μ . The suitcase does not slip, even if angle θ is made slightly larger.

Which expression gives the friction force between the suitcase and the belt?

(gravitational field strength = g)

- A μmg
 B $mg \sin \theta$
 C $mg \cos \theta$
 D $\mu mg \sin \theta$
 E $\mu mg \cos \theta$

Friction = Component of weight
 down slope
always
 $= mg \sin \theta$

- 51 The curve $y = \sin x$ is stretched by a scale factor of $\frac{1}{2}$ parallel to the x -axis and then translated by $\frac{\pi}{4}$ in the negative x -direction.

What is the equation of the new curve?

- A $y = \sin\left(\frac{x}{2} - \frac{\pi}{4}\right)$
 B $y = \sin\left(\frac{x}{2} + \frac{\pi}{4}\right)$
 C $y = \sin\left(\frac{x}{2} - \frac{\pi}{8}\right)$
 D $y = \sin\left(\frac{x}{2} + \frac{\pi}{8}\right)$
 E $y = \sin\left(2x - \frac{\pi}{4}\right)$
 F $y = \sin\left(2x + \frac{\pi}{4}\right)$
 G $y = \sin\left(2x - \frac{\pi}{2}\right)$
 H $y = \sin\left(2x + \frac{\pi}{2}\right)$

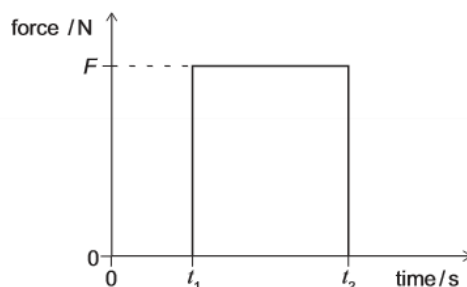
① $y = \sin(2x)$

② $y = \sin\left(2x + \frac{\pi}{2}\right)$



Already scaled
 by $\frac{1}{2}$

- ✗ The graph shows how the horizontal force on a tennis ball of mass m varies during a shot in a tennis match. The ball is initially travelling horizontally toward the racket with speed u and leaves the racket horizontally travelling in the opposite direction with speed v .



Which expression gives the magnitude of the momentum of the ball as it leaves the racket?

- A $F(t_2 - t_1)$
 B $F(t_2 - t_1) - mu$
 C $F(t_2 - t_1) + mu$
 D $mv - mu$
 E $Ft_2 - mu$

$$\text{Impulse} = F(t_2 - t_1)$$

$$m(v + u) = F(t_2 - t_1)$$

$$mv = F(t_2 - t_1) - mu$$

- 53 The equations of two straight lines are $y = 3 + (2p^2 - p)x$ and $y = 7 + (p - 2)x$, where p is a real constant.

For certain values of p , the two lines are perpendicular.

Which of the following numbers is closest to the greatest such value of p ?

- A 2.00
 B 1.75
 C 1.50
 D 1.00
 E -0.25
 F -0.50

$$2p^2 - p = \frac{-1}{p - 2}$$

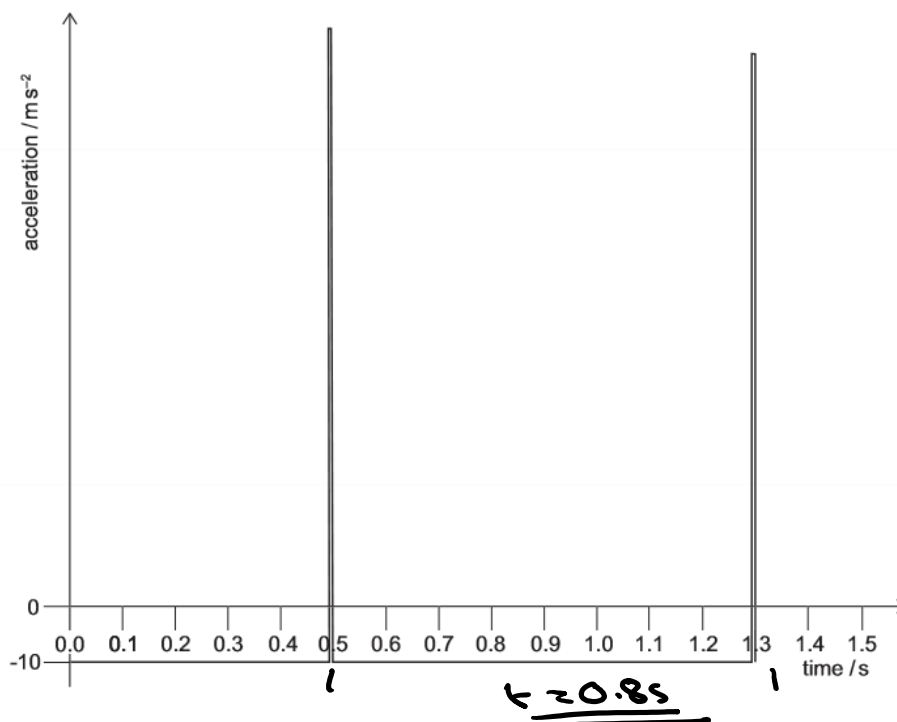
$$(2p^2 - p)(p - 2) + 1 = 0$$

$$2p^3 - 5p^2 + 2p + 1 = 0$$

$$(p - 1)(2p^2 - 3p - 1) = 0$$

$$p = \frac{3 \pm \sqrt{9 - 4(2)(-1)}}{4} = \frac{3 \pm \sqrt{17}}{4} \approx 1.75 \text{ (greatest value)}$$

- ✗ The acceleration versus time graph is for a ball dropped from rest, falling vertically and bouncing on the ground.



The time of contact with the ground can be ignored.

What is the speed of the ball immediately **after** hitting the ground for the first time, and what is the maximum height reached by the ball after the first bounce?

(gravitational field strength = 10 N kg^{-1})

	speed / m s^{-1}	height / m
A	4.00	0.80
B	4.00	1.25
C	5.00	0.80
D	5.00	1.25
E	8.00	3.20

$$\begin{aligned}
 s &= ut + \frac{1}{2}at^2 \\
 0 &= u(0.8) - 5(0.8^2) \\
 u &= \frac{5(0.8^2)}{0.8} = 4 \text{ m s}^{-1} \\
 0 &= 16 - 20s \quad (v^2 = u^2 + 2as) \\
 s &= \frac{16}{20} = 0.8 \text{ m}
 \end{aligned}$$

END OF TEST